Credit Cards and Inflation*

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Abstract

The introduction (and widespread use) of credit cards increases trading efficiency but, by also increasing the velocity of money, it causes massive inflation, in the absence of monetary intervention.

If the monetary authority attempts to restore pre-credit card price levels by reducing the money supply, it will have to sacrifice *all* the efficiency gains.

When there is default on credit cards, there is even more inflation, and less efficiency gain. The monetary authority will then have to accept *less* than pre-credit card efficiency in order to restore pre-credit card price levels, or else it will have to accept *substantial* inflation if it is unwilling to cut efficiency below pre-credit card levels. This is stagflation.

Key Words: Credit cards, outside money, inside money, central bank, inflation, stagflation

JEL Classification: D50, D51, D53, D61, E40, E50, E51, E52, E58

We argue that the introduction and widespread use of credit cards increases trading efficiency but must cause a massive increase in price levels, other things being equal. Government monetary intervention sufficient to stop these price increases must necessarily undo all the efficiency gains that credit cards bring. Things are worse if there is default on credit cards: large price increases are inevitable unless the monetary authority is willing to engineer substantial reductions in trading efficiency.

In modern economies, more and more transactions take place via credit cards. They are perhaps the single most visible and talked about economic innovation in the last 40 years. Yet credit cards have not been extensively studied by general equilibrium theorists or monetary theorists, presumably because it has been thought that the effects of credit cards are negligible, or easily managed by monetary interventions. Insofar as they are mentioned in the modern economics curriculum at all, it is only in the context of calibrating the rationality of consumers, or lamenting the indebtedness of the household sector. The efficiency gains they bring, and their effect

*Both authors have benefited greatly from many years of collaboration and conversation on the theory of money with Martin Shubik. It is a pleasure to dedicate this paper to him.

on price levels, have been ignored. An older macroeconomic literature in the 1950s and 60s did raise these issues about "near monies", but this was before the advent of credit cards, in an intellectual era of reduced form models in which it would have been impossible to directly analyze credit cards anyway.[1]

We introduce a one-period general equilibrium model with money and credit cards in which individual agents choose whether to buy goods with cash or credit cards, and prices adjust in order to clear all markets. No assumptions are needed on the number of commodities or the form of the utilities (beyond the usual general equilibrium hypotheses of continuity and concavity). We show in a series of theorems that a widespread use of credit cards must have a profound effect on price levels in the absence of monetary interventions, and that policy interventions to prevent price increases can be problematic. We do not deal with the transition from the regime without credit cards to the new regime with credit cards, preferring to keep the analysis as simple as possible by restricting ourselves in this paper to a one-period model. In a dynamic economy we would expect to see several periods of rapid inflation after credit cards are introduced, tapering off only when prices settle down at much higher levels.

The surge in price levels in the United States in the 1970s and early 1980s coincided with the introduction of credit cards. Though we are in no position at this time to make an empirical connection between credit cards and the stagflation in the 1970s and early 80s, our model provides a theoretical possibility of a causal connection. Many countries have introduced credit cards at different times over the past 30 years, with differing levels of default. In future work we hope to take advantage of this data to provide an empirical test of the theoretical conclusions we derive here.

The paper is organized as follows. In Section 1 we recall the one-period general equilibrium monetary economy that we ourselves introduced (Dubey-Geanakoplos (1992), (2003)).[2] We could probably have studied credit cards in other monetary general equilibrium models, but ours appears to be the simplest.[3]Our model embodies the distinction Gurley and Shaw emphasized between inside and outside money; this distinction is crucial to the existence of a positive value of money in a one-period economy. Theorem 1 reprises our old observation that monetary equilibrium does exist if there are enough gains to trade available from the initial endowment. The

[1]See for example Gurley and Shaw [1960], Brainard and Tobin [1963], Tobin [1963], and Brainard [1964].

[2]Shubik-Wilson (1973) introduced a central bank loaning inside money, but they did not have individual endowments of outside money. Without default, the bank interest rate must then always be zero and trade must be efficient. In contrast we alllow for individual endowments of money, which leads to a positive bank interest rate and inefficient trade even when there is no default.

[3]The famous Lucas-Stokey [1987] model is a natural candidate. But that model has an infinite number of periods, making calculations unwieldy. Incidentally, Lucas and Stokey do include credit goods (goods that can be bought without cash in advance). But these goods cannot be bought with cash. The choice of whether to buy with cash or credit cards, and how the market gives incentives for each, is central to our model. There is also no default on credit in the Lucas and Stokey model.

model also displays in rudimentary form the trade-off the monetary authority faces between efficiency and inflation.[4]

Credit cards are introduced in Section 2, and for simplicity, we first examine the idealized situation where default does not occur. Consumers choose whether to buy goods with cash or credit cards. Naturally they find it more convenient to use credit cards, raising the question whether money can survive. Indeed many commentators refer to the coming "cashless" economy in which the supply of inside and outside money (i.e., cash) will be irrelevant. It is tempting to think that if credit cards became available to all households for the purchase of all commodities, and if there were no credit limits, then virtually all transactions would be conducted via credit cards, eventually eliminating the use of money altogether. These conclusions depend on how credit cards are settled. In the natural case, corresponding roughly to the situation faced by the typical consumer today (in 2008), credit card debts and receipts are not "netted." A consumer who gets his credit card bill must find the cash to pay, perhaps by writing a check on his bank account or paying with a debit card.[5] He cannot point out that as a merchant he has sold goods to customers charged on their credit cards, who owe him as much money as he himself owes. Without netting, credit cards do not alleviate the need for money, they only postpone it. Thus without netting we are able to show quite generally in Theorem 2 that money remains viable, i.e. that an equilibrium exists in which money has positive (albeit diminished) value.

Theorem 2 shows that credit cards improve trading efficiency. Credit card purchases do crowd out some cash transactions, but they do not threaten the viability of money; indeed they enhance it. By improving the efficiency of transactions, credit cards paradoxically create circumstances or parameter values for which money could not have any positive value on its own, but does after credit cards are added.

Theorem 2 also resolves the puzzle of how cash and credit cards can co-exist. Credit card prices are higher than cash prices, or in other words, cash purchases are made at a discount. (In a multi-period model, equality of cash and credit card prices can be maintained, provided there are consumers who pay interest on their credit card debt).

If credit cards only postpone the need for money, one might wonder whether they have any effect on cash prices. In Section 3 we examine the inflation caused by credit cards, when the monetary authority remains passive, and the potential stagflation when it tries to intervene. We show that credit cards cause a *massive* rise in cash price levels because they allow money to do more work. In Theorem 3 we prove that

[4] Inflation normally means the rate of increase of price levels over time. This cannot be modeled in our one-period economy. So by inflation we mean the comparative statics exercise in which price levels rise.

[5] In the model we shall shortly describe, we allow for cash and for credit cards, but not for example for debit cards or checking accounts. These extra instruments are similar to cash; indeed their differences from cash can only be rigorously modeled in a multiperiod setting. In any case, with or without them, the issues connected with credit card purchases are quite similar, so we treat those issues in the simplest setting, without debit cards or checking accounts.

when there is no default or credit limits in a one-period economy, aggregate cash expenditures necessarily equal aggregate credit card expenditures. Theorem 4 shows that the introduction of credit cards creates a new equilibrium in which the price level is higher, on the order of about 100%. Furthermore, their introduction has the identical effect on cash prices and commodity allocations as the infusion of vastly more inside money. (We compute precisely how much).

A monetary authority, alarmed by the inflation, might try to undo these effects by tightening the money supply. We show in Theorem 5 that the authority can indeed cut the money supply to reproduce the pre-credit card equilibrium cash prices. But at the same time it will have to reduce trade to the pre-credit card equilibrium levels. This means giving up *all* the efficiency gains created by the credit cards.

Furthermore, this tightening does not completely undo the inflation because credit cards and money are not perfect substitutes: if the cash prices are brought back to their pre-credit card levels, the credit card prices will have to be slightly higher. Thus the average (credit card and cash) price would not be restored. Curiously, the expansion of the feasible set, due to the improvement in transactions technology brought on by credit cards, leads to a *worse* outcome: the same trades but higher prices or, with more tightening, less real trades and slightly higher average prices. This is a touch of stagflation.

In Section 4 we work out an example illustrating Theorems 1-5.

In Section 5 we introduce credit card default, which is so prevalent in the real economy, via a simple example. We imagine a world of reliable Dr. Jekylls who always keep their credit card promises and unreliable Mr. Hydes who never do. Through credit card default the Hydes are able to spend more money than they have. Of course in equilibrium everyone realizes this, and the default rate on credit cards is rationally anticipated.

We show that default accentuates the inflation brought on by credit cards and, despite the extra demand from the Hydes, it lowers the volume (efficiency) of trade, leaving it between the pre-credit card level and the level with credit cards that don't default. Utility of the honest Jekylls is worse than it was in the pre-credit card equilibrium.

Thus credit cards can be bad for the economy when default occurs. They can create a gigantic inflation and at the same time reduce the utility of honest Jekylls. If the monetary authority, alarmed by this inflation, tries to correct the situation by reducing the money supply, it will make things worse. The volume of trade will fall, as will utilities. At the point where price levels are restored all the way back to their pre-credit card levels, the volume of trade will be lower than its pre-credit card level, and the utilities of the Jekylls will be much worse. The monetary authority might very well stop short of reducing money supply so far, perhaps stopping when the volume of trade is at pre-credit card levels. But this would leave prices above their their pre-credit card levels; in our example, 24% higher. Most likely, the monetary authority will stop somewhere in between, creating a new equilibrium with higher prices, less volume of trade, and lower utilities. Thus with default on credit cards we get not just a touch of stagflation, but robust stagflation.

In Section 6 we introduce a general model of default with nominal penalties. Default would seem to be a disequilibrium phenomenon, but we show that there is no problem reconciling it with equilibrium. We allow households to choose to default on their credit card payments. Those who do must pay a penalty. If the penalty is very high, nobody will default. But for intermediate default penalties, some households will default and credit card receipts will correspondingly decline. Credit card prices will respond by rising relative to cash prices, reflecting a default premium. Those who keep their credit card promises effectively pay for those who default. As the penalty falls, there will come a catastrophic point beyond which credit cards will be abandoned and the economy will revert to exclusively using cash. Theorem 6 guarantees that whatever the level of credit card default penalties, credit card equilibrium will nonetheless exist (with or without the use of credit cards), under the same conditions as in Theorem 1.

In Section 6 we show that the various phenomena exhibited in our example of Section 5 continue to hold in the general model. Section 6.6.1 elaborates the example from Section 5 in the context of default. It also briefly explores the existence of monetary equilibrium when netting is permitted on credit cards. All proofs are relegated to Section 7.

In this paper we confine ourselves to a single period setting. In forthcoming work (Geanakoplos-Dubey (2007)) we examine a multi-period model, and we show that financial innovations such as credit cards have a much bigger effect. In a one-period model households will not hold idle money, but in a multi-period world without credit cards, agents do hold idle cash. The introduction of credit cards then puts the idle money to work as well as increasing the velocity of the spent money, thus leading to still higher prices.

Furthermore, a naive central bank, that tries to hold prices at their pre-credit card levels by increasing interest rates, will disproportionately hurt long-run producers who do not benefit from the credit cards but do suffer from the higher interest rates. The overreaction of the central bank could *substantially cut production* below pre-credit card levels, possibly triggering substantial stagflation (without any defaults on credit cards). A dynamic model also permits us to study the inflationary transition from a cash economy to a credit card economy. One possibility is that as the use of credit cards gradually spreads throughout the population, most of the inflation comes at the beginning, engendered sheerly by the rational anticipation of the full spread of credit cards.

1 The Pure Monetary Economy

Before introducing credit cards in Section 2, we recall the model from Dubey-Geanakoplos (1992, 2003) with money as the sole medium of exchange.

1.1 The Underlying Economy

We consider a standard pure exchange general equilibrium economy which lasts just one time period and has only private goods (commodities) $L = \{1, ..., L\}$. The agents in the economy are households $H = \{1, ..., H\}$. Each $h \in H$ has an endowment of commodities $e^h \in \mathbb{R}^L_+$ and a utility of consumption $u^h : \mathbb{R}^L_+ \to \mathbb{R}$. We assume: (a) $e^h \neq 0$ for all $h \in H$, i.e., every household has at least some endowment (e.g., his own labor); (b) $\sum_{h \in H} e^h \gg 0$, i.e., every named commodity is present in the aggregate; (c) u^h is continuous, concave, and strictly increasing in each[6] variable, for all $h \in H$. The underlying economy, which constitutes the real sector of our model, is denoted $\mathcal{E} \equiv (u^h, e^h)_{h \in H}$.

1.2 Money and Bank Loans

Money is stipulated to be the *sole* medium of exchange, until credit cards are introduced. It is fiat and gives no direct utility of consumption to the households; they value money only insofar as it enables them to acquire commodities for consumption.

Money enters the economy in two ways: as private endowment $m^h \geq 0$ of household $h \in H$ and as a stock $M > 0$ at a bank. Apart from households, the bank is the only other agent in our model, but it has a passive role. It stands ready to lend M to households at an interest rate that is determined endogenously in equilibrium. Both $m \equiv \{m^h\}_{h \in H}$ and M are exogenously fixed as part of the data of the model. The sum $\bar{m} \equiv \sum_{h \in H} m^h > 0$ constitutes the stock of *outside money*, which households own free and clear of debt, at the start of the economy; while the bank stock M is *inside money* and is always accompanied by debt when it comes into households' hands. We denote the monetary economy by $(\mathcal{E}, m, M) \equiv ((u^h, e^h, m^h)_{h \in H}, M)$; and its private sector by $(\mathcal{E}, m) \equiv (u^h, e^h, m^h)_{h \in H}$.

The period is divided into three time intervals. In the first interval, households borrow money from the bank. In the second interval, they exchange commodities for money and vice versa. In the third interval, they repay bank loans with money and consume. Default is not permitted.

The interest rate r and the commodity prices p are regarded as fixed by the households before they trade, although as is usual in general equilibrium, it is their trades which determine these prices. In the first interval households sell IOU notes or bonds promising \$1 to the bank in exchange for cash of $1/(1 + r)$ dollars. In the second interval all commodity markets meet simultaneously. There is a cash-in-advance constraint which requires households to pay money to purchase commodities at prices p at the different markets. It is only in the third interval, after these markets close, that revenue from the sales of commodities comes into households' hands, by which time it is too late to use this revenue for purchases. Those households who

[6] *Strict* monotonicity is assumed for ease of presentation, and can be weakened (see Dubey-Geanakoplos [2003]).

find their endowment m^h of money insufficient will need to borrow money from the bank to finance purchases, and will defray the loan out of their sales revenue.

1.3 Prices

We denote money by m (without confusing it with the vector $m \equiv (m^1, ..., m^H)$ of household endowments). Let $p_{\ell m} > 0$ denote the price of commodity $\ell \in L$ in terms of money, and let $r \geq 0$ denote the interest rate on the bank loan. The vector $(p, r) \in \mathbb{R}^L_{++} \times \mathbb{R}_+$ will be referred to as "market prices."

1.4 The Budget Set of a Household

Since money is the sole medium of exchange, the vector q^h of market actions of household h has $2L+1$ components (where $\ell \in L$ and n denotes an IOU or bond sold to the bank):

$q^h_{nm} \equiv$ quantity of bonds sold by h to the bank for money
$q^h_{m\ell} \equiv$ money spent by h to purchase ℓ
$q^h_{\ell m} \equiv$ quantity of ℓ sold by h for money

(It is evident, on account of there being just one period, that no household would improve its consumption by depositing money at the bank to earn interest. So, we suppress deposits, i.e., the purchase of bonds q^h_{mn}.)

We consider the case of a perfectly competitive household sector. Each $h \in H$ regards market prices $(p, r) \in \mathbb{R}^L_{++} \times \mathbb{R}_+$ as fixed (uninfluenced by its own actions). The *budget set* $B^h(p, r)$ consists of all market actions and consumptions $(q^h, x^h) \in \mathbb{R}^{2L+1}_+ \times \mathbb{R}^L_+$ that satisfy the budget constraints (1), (2), (5), and (3ℓ), (4ℓ), (6ℓ) listed below, for all $\ell \in L$. The residual variables $\tilde{x}^h = \tilde{x}^h(q^h, p)$ and $\tilde{m}^h = \tilde{m}^h(q^h, r)$ are determined automatically by q^h, p, r.

$$\tilde{m}^h \equiv \frac{q^h_{nm}}{1+r} \tag{1}$$

$$\sum_{\ell \in L} q^h_{m\ell} \leq m^h + \tilde{m}^h \tag{2}$$

$$q^h_{\ell m} \leq e^h_\ell \tag{3ℓ}$$

$$\tilde{x}^h_\ell \equiv \frac{q^h_{m\ell}}{p_{\ell m}} \tag{4ℓ}$$

$$q^h_{nm} \leq \Delta(2) + \sum_{\ell \in L} p_{\ell m} q^h_{\ell m} \tag{5}$$

$$x^h_\ell \leq (\Delta 3\ell) + \tilde{x}^h_\ell. \tag{6ℓ}$$

Here $\Delta(\alpha)$ is the difference between the right and left sides of inequality (α). The interpretation is clear: (1) says that household h borrows $\tilde{m}^h$ dollars by promising to pay $q^h_{nm} = (1+r)\tilde{m}^h$ dollars after commodity trade; (2) says that total money spent on purchases cannot exceed the money on hand, i.e., money endowed plus money borrowed; (3ℓ) says that no household can sell more of any commodity than it is endowed with; (4ℓ) says that households purchase commodities with money at market prices p; (5) says that we are not permitting default, i.e., every household must fully deliver on its bonds; (6ℓ) says that the consumption vector cannot exceed what a household winds up with after trade.

1.5 Monetary Equilibrium

A vector of prices and household actions

$$\langle (p,r), (q^h, x^h)_{h\in H} \rangle$$

is a *monetary equilibrium* (ME) of $(\mathcal{E}, m, M)$ if all households optimize in their budget sets, i.e.,

$$\begin{aligned} &\text{(a) } (q^h, x^h) \in B^h(p,r) \\ &\text{(b) } u^h(x^h) \geq u^h(\underline{x}^h) \text{ for all } (\underline{q}^h, \underline{x}^h) \in B^h(p,r); \end{aligned} \tag{7}$$

and demand equals supply for all commodity markets and the bank loan market, i.e.,

$$\begin{aligned} &\text{(a) } \sum_{h\in H} \tilde{x}^h_\ell = \sum_{h\in H} q^h_{\ell m},\ \ell \in L \\ &\text{(b) } \sum_{h\in H} \tilde{m}^h = M. \end{aligned} \tag{8}$$

Notice that since the components of p at any ME must be finite by definition, money will have positive value at an ME.

It is worth noting that in a monetary equilibrium, the total stock of money and commodities held in the hands of the bank and the households is conserved in all three time intervals into which the period is divided. In the beginning of the first interval the bank holds M and households hold $\bar{m}$ of money. At the end of the first interval and in the beginning of the second interval households hold all of $M+\bar{m}$. The money is rearranged among the households in the second interval, so all of $M+\bar{m}$ is still in their hands at the end of the second interval and at the beginning of the third interval. At the end of the third interval, the bank is left holding all of $M+\bar{m}$. The reason for the latter is that no household will be left with unowed cash after repaying the bank, otherwise he should have borrowed and spent more money earlier to purchase commodities, or else curtailed his sale of commodities, improving his utility. Thus all of $M+\bar{m}$ does end up with the bank. But that would not happen unless at least $M+\bar{m}$ is owed to the bank. Since default is not permitted, no more could be owed to the bank. Thus $(1+r)M = M+\bar{m}$ at any ME, i.e., $r = \bar{m}/M$.

1.6 Gains to Trade

Money is needed only for trading commodities. It follows that the value of money in the economy must depend entirely on the level of households' *a priori* motivation to trade commodities with each other. What is needed is an intrinsic "gains-to-trade" hypothesis. We developed a measure of gains-to-trade in Dubey-Geanakoplos [1992,2003], and showed that, whenever they are strong enough from the initial endowment, monetary equilibrium exists, i.e. money has value.

The idea is to check whether a central planner with a "γ-handicap" can nevertheless reallocate goods to make everybody better off. After the planner collects commodities from the agents, he must throw away a fraction $\gamma/(1+\gamma)$ of everything collected and then redistribute what remains.

To make this precise, let $\tau^h \in \mathbb{R}^L$ be a trade vector of h (with positive components representing purchases and negative components representing sales). For a scalar $\gamma > -1$, define

$$\tau_\ell^h(\gamma) = \min\{\tau_\ell^h, \tau_\ell^h/(1+\gamma)\}$$

Note $\tau_\ell^h(\gamma) = \tau_\ell^h$ if $\tau_\ell^h < 0$, $\tau_\ell^h(\gamma) = \tau_\ell^h/(1+\gamma)$ if $\tau_\ell^h > 0$. Thus $\tau^h(\gamma)$ entails a diminution of purchases in τ^h by the fraction $\gamma/(1+\gamma)$.

We say that there are *gains-to-γ-diminished-trade* from $x \equiv (x^h)_{h\in H} \in (\mathbb{R}_+^L)^H$ if there exist trades $(\tau^h)_{h\in H}$ such that:

(a) $\sum_{h\in H} \tau^h = 0$

(b) $x^h + \tau^h \in \mathbb{R}_+^L$ for all $h \in H$

(c) $u^h(x^h + \tau^h(\gamma)) > u^h(x^h)$ for all[7] $h \in H$.

We define $\gamma(x)$ as the least upper bound of all handicaps that permit Pareto improvement.

Definition: The **gains-to-trade from $\boldsymbol{x}$** are given by

$$\gamma(x) = \sup\{\gamma : \text{there are gains-to-}\gamma\text{-diminished-trade from } x\}.$$

The number $\gamma(x)$ can be viewed as a local measure of the departure from Pareto optimality. It represents how easy it is to make a small Pareto improvement starting from x. If x is Pareto optimal, then $\gamma(x) = 0$; otherwise $\gamma(x) > 0$. With two agents and two goods, $[1+\gamma(x)]^2$ is the ratio of the agents' marginal rates of substitution for the two goods.

We quote the following result from Dubey-Geanakoplos (2003).

Theorem 1 C*onsider a monetary economy* $(\mathcal{E}, m, M)$ *in which* $\gamma(e) > \bar{m}/M$. *Then a monetary equilibrium exists. At any monetary equilibrium* $(p, r, (q^h, x^h)_{h\in H})$, *the interest rate* $r = \bar{m}/M$ *and the value of aggregate trade is* $\sum_{\ell\in L}\sum_{h\in H} p_{\ell m} q_{\ell m}^h =$

[7] Since utilities are strictly monotonic, this is equivalent to requiring that some household is strictly better off and none are worse off.

$M+\bar{m}$. If the utilities $(u^h)_{h\in H}$ are smooth, then the unexploited gains to trade from the equilibrium allocation $\gamma((x^h)_{h\in H}) = \bar{m}/M$.

Moreover, for generic smooth utilities, endowments, and money stocks, the economy $((u^h, e^h, m^h)_{h\in H}, M)$ has finitely many monetary equilibria.

Note that if $\bar{m} > 0$, monetary equilibrium is never Pareto optimal for smooth economies, since $\gamma((x^h)_{h\in H}) > 0$.

2 The Credit Card Economy

Let us now imagine that credit cards are introduced into our monetary economy. Households can buy commodities directly with the credit cards, without having to borrow any money in advance. Of course the seller then gets a promise, and not cash, for his good. The simplest timing is to suppose that credit card purchases are made simultaneously with cash purchases, but that credit card debts must be repaid just before bank loans come due.

Denote by c the promise of one unit of money via the credit card. For $\ell \in L$, let

$p_{\ell c} \equiv$ credit card price of $\ell \equiv$ price of ℓ in terms of c

Note that the cash price $p_{\ell m}$ of a commodity need not be the same as its credit card price $p_{\ell c}$. By selling for cash, one gets the money sooner. So it might well be that $p_{\ell m} < p_{\ell c}$. (In practice goods can often be purchased at a discount with cash.) Thus market prices are now given by a longer vector $(p, r) \equiv ((p_{\ell m})_{\ell\in L}, (p_{\ell c})_{\ell\in L}, r)$ with $2L + 1$ components.

We denote the credit card economy by $(\mathcal{E}^c, m, M)$.

The vector q^h of market actions of household h now has[8] $2L+2L+1$ components, with

$q^h_{\alpha\beta} \equiv$ quantity of α sent by h to the market $\alpha\beta$

where the markets are nm (the bank loan market), $(\ell c)_{\ell\in L}$ (the credit card-commodity markets), $(\ell m)_{\ell\in L}$ (the cash-commodity markets).

2.1 Credit Card Budget Set

Given (p, r), the vector (q^h, x^h) must satisfy:

$$\tilde{m}^h \equiv \frac{q^h_{nm}}{1+r} \tag{1*}$$

$$\sum_{\ell\in L} q^h_{m\ell} \le m^h + \tilde{m}^h \tag{2*}$$

$$q^h_{\ell c} + q^h_{\ell m} \le e^h_\ell \tag{3*ℓ}$$

[8] Since bank deposits are returned after clearing debts on the credit card, once again no household will want to deposit money at the bank, and we suppress deposits.

$$\tilde{x}_\ell^h(m) \equiv \frac{q_{m\ell}^h}{p_{\ell m}},\ \tilde{x}_\ell^h(c) \equiv \frac{q_{c\ell}^h}{p_{\ell c}} \tag{4*ℓ}$$

$$\sum_{\ell \in L} q_{c\ell}^h \leq \Delta(2^*) + \sum_{\ell \in L} p_{\ell m} q_{\ell m}^h \tag{5*}$$

$$q_{nm}^h \leq \Delta(5^*) + \sum_{\ell \in L} p_{\ell c} q_{\ell c}^h \tag{6*}$$

$$x_\ell^h \leq \Delta(3\ell^*) + \tilde{x}_\ell^h(c) + \tilde{x}_\ell^h(m) \tag{7*ℓ}$$

The budget set $B_c^h(p, r)$ of household h consists of all (q^h, x^h) that satisfy constraints (1^*) to (7^*). (1^*) and (2^*) are as in the monetary economy. $(3^*\ell)$ says that household h sells good ℓ separately against cash and credit cards, but cannot sell in total more than it has. $(4^*\ell)$ says that household h buys good ℓ separately with cash and credit cards. (5^*) requires that credit card debts be paid in full with money, before cash receipts from credit card sales become available, and before bank loans come due. (6^*) says that after receiving all the money from sales against credit cards, there is enough money to pay off all bank loans. $(7^*\ell)$ constrains household h to consume no more of commodity ℓ than it has after trade.

The critical thing to notice is that the same dollar can be used to repay two different debts: one household uses the dollar to pay his credit card debt, and the recipient uses the same dollar to pay his bank loan.

Thus credit cards enable money to do extra work. This inevitably causes inflation, as we shall see shortly. In a multiperiod model (see Geanakoplos-Dubey (2006)), inflation is even higher because households drastically reduce their holdings of idle cash, making money work still harder.

2.2 Credit Card Equilibrium

We say that $\langle p, r, (q^h, x^h)_{h \in H} \rangle$ is a credit card equilibrium of the economy $(\mathcal{E}^c, m, M)$ if:

$$\text{(a) } (q^h, x^h) \in B_c^h(p, r) \tag{8*}$$

$$\text{(b) } u^h(x^h) \geq u^h(\underline{x}^h) \text{ for all } (\underline{q}^h, \underline{x}^h) \in B_c^h(p, r); \tag{7}$$

for all $h \in H$, i.e., all agents optimize on their budget sets and

$$\text{(a) } \sum_{h \in H} \tilde{m}^h = M \tag{9*}$$

$$\text{(b) } \sum_{h \in H} \tilde{x}_\ell^h(m) = \sum_{h \in H} q_{\ell m}^h \quad \forall \ell \in L \tag{7}$$

$$\text{(c) } \sum_{h \in H} \tilde{x}_\ell^h(c) = \sum_{h \in H} q_{\ell c}^h \quad \forall \ell \in L \tag{1}$$

i.e., all markets clear.

2.3 Existence and Efficiency of Credit Card Equilibrium

Theorem 2 *Let $(\mathcal{E}^c, m, M)$ be a credit card economy and assume that $\gamma(e) > \sqrt{1 + \frac{\bar{m}}{M}} - 1$. Then a credit card equilibrium exists. Moreover, if $\langle p, r, (q^h, x^h)_{h \in H} \rangle$ is a credit card equilibrium, then we must have $r = \bar{m}/M$, and $p_{\ell c} = \left(\sqrt{1 + \frac{\bar{m}}{M}} \right) p_{\ell m}$ for all $\ell \in L$. Finally, if the utilities $(u^h)_{h \in H}$ are smooth, then the unexploited gains to trade $\gamma((x^h)_{h \in H}) = \sqrt{1 + \frac{\bar{m}}{M}} - 1$.*

The proof of Theorem 2 is contained in the proof of Theorem 4 (given in Section 7).

Since $\frac{\bar{m}}{M} > 0$, it follows that $\sqrt{1 + \frac{\bar{m}}{M}} < 1 + \frac{\bar{m}}{M}$, so that credit card equilibrium exists whenever monetary equilibrium exists, and continues to exist when $\sqrt{1 + \frac{\bar{m}}{M}} < 1 + \gamma(e) < 1 + \frac{\bar{m}}{M}$, even where monetary equilibrium might not. The introduction of credit cards thus enhances the viability of money.

It is often said that credit cards will drive out money, and that we are headed toward a cashless economy. Observe, however, that the reasons which make credit card purchases more attractive than cash purchases to the buyer often make them less attractive to the seller. Buyers prefer to pay later rather than earlier, and a credit card purchase enables them to defer the transfer of cash. But for precisely this reason, sellers prefer cash buyers. Similarly, as we shall see in Section 6, a buyer might prefer to purchase by promise, retaining the option of defaulting if his circumstances do not improve, and once again sellers prefer ready cash to promises that may be broken. The presumption that credit cards must eventually drive out cash neglects half the market, since it ignores the sellers.

The key is to recognize that cash and credit cards will coexist if cash prices are lower than credit card prices.

Notice that in our model the introduction of credit cards does not affect the bank rate of interest $r = \bar{m}/M$. Nevertheless, since the gains to trade remaining at credit card equilibrium are lower than the gains to trade remaining at monetary equilibrium, credit cards lead to more efficient trade. This is because the "effective" rate of interest paid in credit card equilibrium is $\sqrt{1 + \bar{m}/M} - 1$, which is lower than the rate $\bar{m}/M$ prevailing in pure monetary equilibrium.

In the pure monetary economy, households borrowed money from the bank to purchase commodities and sold commodities for cash in order to defray the bank loan. As a result, buyers who borrowed a dollar to spend on goods had to sell goods worth $(1 + r)$ dollars in order to repay the bank. The introduction of credit cards enables households to reduce this wedge to $\sqrt{(1 + r)}$ by engaging in either of two equivalent trading strategies. From Theorem 2, credit card prices are precisely $\sqrt{(1 + r)}$ higher than cash prices. Any household can purchase commodities, whose cash prices are \$1, by charging $\$\sqrt{(1+r)}$ on a credit card and then selling goods for cash worth $\$\sqrt{(1+r)}$ to defray the credit card debt, indeed reducing the wedge to $\sqrt{(1+r)}$. Or else, he can borrow \$1 from the bank, spend the cash on goods, while simultaneously raising enough money to repay the bank by selling other commodities against credit cards for $\$(1+r)$. Since the credit card sales have cash value equal to $\$(1+r)/\sqrt{(1+r)} = \$\sqrt{(1+r)}$, the wedge is again $\sqrt{(1+r)}$.

We shall call these two trading strategies *buy-credit/sell-cash and buy-cash/sell-credit*, respectively. They are reflected in (i) and (ii) of Theorem 3 below.

2.4 Flow of Funds in Credit Card Equilibrium

Theorem 3 *Let $(\mathcal{E}^c, m, M)$ be a credit card economy and assume that $\langle p, r, (q^h, x^h)_{h\in H}\rangle$ is a credit card equilibrium. Then (i) individual credit card debt equals individual cash receipts, i.e., $\sum_{\ell=1}^{L} q_{c\ell}^h = \sum_{\ell=1}^{L} p_{\ell m} q_{\ell m}^h$, and so aggregate credit card debt is equal to aggregate cash receipts, i.e, $\sum_{h=1}^{H}\sum_{\ell=1}^{L} q_{c\ell}^h = \sum_{h=1}^{H}\sum_{\ell=1}^{L} p_{\ell m} q_{\ell m}^h$. Similarly, (ii) individual bank debt equals individual credit card receipts, i.e. $q_{nm}^h = \sum_{\ell=1}^{L} p_{c\ell} q_{\ell c}^h$, and so aggregate bank debt equals aggregate credit card receipts $\sum_{h=1}^{H} q_{nm}^h = \sum_{h=1}^{H}\sum_{\ell=1}^{L} p_{c\ell} q_{\ell c}^h$.*

The proof of Theorem 3 also follows from the proof of Theorem 4.

Households sell commodities for cash (at prices lower than they could get by selling against credit cards) only in order to defray their own credit card debt, incurrred in the course of following the first trading strategy described above. This is the content of (i) of Theorem 3.

Since all the sales revenue from the commodity-cash markets are used to redeem credit card debts, households must repay their bank loans out of the sales revenue from commodity-credit card markets. This explains (ii) of Theorem 3.

Since total cash receipts must equal total cash expenditures, and since in a one-period model all cash is spent, both must equal $(M+\bar{m})$. From (i) we conclude that aggregate credit card debt is equal to $(M+\bar{m})$. But credit card debt is another word for credit card expenditures, hence we have

Corollary to Theorem 3: *In any credit card equilibrium, total expenditures on cash markets = total expenditures on credit card markets (albeit calculated at different prices). In short, $\sum_{h=1}^{H}\sum_{\ell=1}^{L} q_{c\ell}^h = \sum_{h=1}^{H}\sum_{\ell=1}^{L} q_{m\ell}^h = (M+\bar{m})$. Thus total expenditures are $2(M+\bar{m})$.*

The introduction of credit cards in our one-period model doubles expenditures, independent of the interest rate or preferences of the agents. Since aggregate cash expenditures are the same as aggregate credit card expenditures, but credit card prices are uniformly higher than cash prices, it follows that more than half of all sales are against cash. Credit card purchases indeed crowd out cash purchases, but never more than half (in our one-period model).

3 Credit Cards and Inflation

The main effect of credit cards is to increase prices, i.e, they lower the value of money even as they increase its viability. The following theorem shows that the introduction of credit cards is tantamount to an infusion of a huge amount of bank money. As was said, its proof, in conjunction with Theorem 1, also yields a constructive proof of Theorem 2, as well as a proof of Theorem 3.

Theorem 4 *Consider a credit card economy* $(\mathcal{E}^c, m, M)$. *Let* $(\mathcal{E}, m, M^*)$ *be a pure monetary economy with more bank money*

$$M^* \equiv M + \sqrt{M^2 + \bar{m}M}$$

Then the equilibria of the two economies coincide in the following sense. For every credit card equilibrium $(p, r, (q^h, x^h)_{h\in H})$ *of* $(\mathcal{E}^c, m, M)$, *there exists a pure monetary equilibrium* $(p^*, r^*, ({}^*q^h, {}^*x^h)_{h\in H})$ *of* $(\mathcal{E}, m, M^*)$ *with the same consumption* $({}^*x^h)_{h\in H} = (x^h)_{h\in H}$ *and the same cash prices* $(p^*_{\ell m})_{\ell\in L} = (p_{\ell m})_{\ell\in L}$ *but a lower interest rate* $(1 + r^*) = \sqrt{1+r}$. *And vice versa.*

Theorem 4 (in conjunction with Theorem 1) yields

Corollary 1 to Theorem 4 *For generic smooth utilities, endowments, and money stocks, the credit card economy* $(\mathcal{E}^c, m, M) = ((u^h, e^h, m^h)_{h\in H}, M)$ *has finitely many credit card equilibrium allocations and prices.*

Note that determinacy is claimed here for equilibrium outcomes, not actions. Typically it will be possible to shift some households' trade from credit card purchases/cash sales into cash purchases/credit card sales, while moving other households in the reverse direction, without disturbing the equilibrium.

Corollary 2 to Theorem 4 *Let* $\mathcal{E}$ *be a smooth underlying economy with a unique Walrasian equilibrium. Let* m *be fixed. Then for all sufficiently large* M, *any pure monetary equilibrium of* $(\mathcal{E}, m, M)$ *has prices nearly proportional to Walrasian, and trades nearly equal to Walrasian. When credit cards are added to* $(\mathcal{E}, m, M)$, *prices will nearly double, without much change in the trade.*

According to the Corollary of Theorem 3, the introduction of credit cards literally doubles the spending (via cash and credit cards) on traded goods. Part of the increase in the money value of trade after credit cards are introduced is due to the increased real trade permitted by more efficient exchange. But the great bulk of the increase comes from higher prices. When $\bar{m}/M$ is very low, as in the scenario of Corollary 2 to Theorem 4, monetary equilibrium trades are necessarily close to Walrasian (and, since the wedge is lower, even closer after credit cards are introduced). Thus almost the entire increase in spending is on exactly the same trades, causing prices to double. Credit cards cause inflation. In a multi-period model the inflation would be even higher, because people would spend their idle cash once they had credit cards to rely on.

3.1 A Touch of Stagflation

We have seen that credit cards increase the efficiency of trade but cause massive inflation. It is natural to imagine a monetary authority that would try to stem this inflation by tightening the money supply and raising interest rates.

What is surprising is that in order to restore the old cash prices, it is necessary to abandon all the gains to trade engendered by the credit cards. In fact, strictly speaking, since the credit card prices are higher than the cash prices, it is actually necessary to reduce trade *below* the original pre-credit card levels in order that the average cash/credit card price be no higher than before. Curiously, a financial innovation (like credit cards) that creates the potential for more efficient trade might end up reducing trade if the monetary authority is committed to preventing *all* inflation.

A conservative monetary authority might well compromise by tolerating a small increase in average prices. But if the increase were small enough, there would necessarily be a drop in efficiency. This is stagflation, though perhaps just a semblance.

Theorem 5 follows immediately from Theorem 4 (taking $M^* = M$ and $M = \hat{M}$) and Theorem 2.

Theorem 5 *Consider a monetary economy* $(\mathcal{E}, m, M)$ *with an equilibrium* $(p, r, (q^h, x^h)_{h\in H})$. *If credit cards are added to the economy, there is always a reduction in the bank money supply to* $\hat{M} < M$ *solving* $M = \hat{M} + \sqrt{\hat{M}^2 + \bar{m}\hat{M}}$ *such that the credit card economy* $(\mathcal{E}^c, m, \hat{M})$ *has an equilibrium* $(\hat{p}, \hat{r}, (\hat{q}^h, \hat{x}^h)_{h\in H})$, *where consumption is what it was before credit cards,* $(\hat{x}^h)_{h\in H} = (x^h)_{h\in H}$, *and cash prices are restored to their pre-credit card levels,* $\hat{p}_{\ell m} = p_{\ell m}$, *but credit card prices are higher than cash prices,* $\hat{p}_{\ell c} = (\sqrt{1 + (\bar{m}/\hat{M})})\hat{p}_{\ell m} > p_{\ell m}$ *for all* $\ell \in L$. *A further reduction in money supply to* $\tilde{M}$ *will lower average prices in the credit card equilibrium* $(\tilde{p}, \tilde{r}, (\tilde{q}^h, \tilde{x}^h)_{h\in H})$ *to their pre-credit card levels, but (assuming utilities are smooth), at the cost of leaving more unexploited gains to trade than there were before credit cards were introduced* $\gamma((\tilde{x}^h)_{h\in H}) > \gamma((x^h)_{h\in H})$.

In our forthcoming paper Geanakoplos-Dubey (2007) we show that this stagflation phenomenon is greatly exacerbated in a multi-period world. But even in our one-period setting, we shall see that we can get genuine stagflation, not just a semblance of it, if there is default on credit cards. The reason is that the default can eliminate most of the efficiency gains of credit cards, while at the same increasing the inflation caused by credit cards. A monetary authority that tries to cut inflation will have to *reduce* real trade.

4 Examples

4.1 Pure Monetary Equilibrium

Consider an exchange economy with two agents and two goods. Let the agents have identical utilities

$$u^h(x_1, x_2) = \log x_1 + \log x_2, \qquad h = 1, 2$$

Let

$$e^1 = (3, 1)$$
$$e^2 = (1, 3)$$

In Walrasian equilibrium, we would have $p = (\lambda, \lambda)$ for any $\lambda > 0$, and $x^1 = x^2 = (2, 2)$. As is well known, Walrasian economics ignores fiat money, and therefore cannot pin down the price *level*; nor does it make room for the nominal rate of interest r.

We first add inside money, then outside money, and finally credit cards to the model.

Suppose $M = 2$ and $m^1 = m^2 = 0$. Then it is easy to see that one monetary equilibrium goes like this: each agent borrows \$1 from the bank at interest rate zero; commodity prices are $p_{1m} = p_{2m} = 1$; agent h spends \$1 buying one unit of commodity $-h \neq h$, while simultaneously selling one unit of commodity h, for $h = 1, 2$; each h takes the \$1 he got from selling and uses it to repay his bank loan.

There is no equilibrium in which the rate of interest $r > 0$. For then the bank would be owed $M(1+r)$, but with only M dollars in existence, there would necessarily be default. There are however many other equilibria with $r = 0$ in which $(p_{1m}, p_{2m}) = \lambda(1, 1)$ with $0 < \lambda \leq 1$. Each agent h borrows \$1 as before, sells one unit of good h, and spends $\$\lambda$ on the other good (good $-h$). He simply hoards the other $\$(1-\lambda)$ returning it unspent, together with the $\$\lambda$ obtained from selling one unit of h. Thus adding inside money M to the economy only reproduces the Walrasian indeterminancy of equilibrium price levels, with an upper bound on λ.

Monetary equilibrium becomes more interesting once we take $m_1 + m_2 > 0$. Then we get a unique equilibrium and positive interest rate, provided the gains to trade at the initial endowment e exceed $(m_1 + m_2)/M$. Let us suppose that $m^1 = m^2 = 1$, and that $M > 1$ is arbitrary.[9]

[9] The gains to trade at e can be immediately computed from the formula in Dubey-Geanakoplos (2003)as $\sqrt{3/1 \cdot 3/1} = 1 + \gamma(e)$, so $\gamma(e) = 2$. Thus if $m^1 + m^2 = 2$, monetary equilibrium exists for all $(m_1 + m_2)/M > 2$, i.e., for $M > 1$.

The stock of inside bank money M, and the outside money (m^1, m^2), in conjunction with the real economy, will determine price levels and the rate of interest. In fact the equilibrium rate of interest must be

$$r = \frac{m^1 + m^2}{M} = \frac{2}{M}$$

By symmetry, we have $x^1 = (3 - \tau, 1 + \tau)$, $x^2 = (1 + \tau, 3 - \tau)$ for some $0 < \tau \leq 1$. Since there is only one period, agents will spend all their money. Hence, once again by symmetry, we must have

$$p_m = p_{1m} = p_{2m} = \frac{M + m^1 + m^2}{2\tau} = \frac{M + 2}{2\tau}$$

Finally, in equilibrium we must have that the ratio of the marginal utilities of consumption are equal to the ratio of prices, distorted by the interest rate wedge. Agent 1 can borrow $p_{2m}\varepsilon$ dollars more and buy ε units of good 2, but then he must sell $(1+r)p_{2m}\varepsilon/p_{1m} = (1+r)\varepsilon$ units of good 1 to raise the revenue to repay the bank. Hence we must have:

$$\frac{3 - \tau}{1 + \tau} = 1 + r.$$

Solving for τ in terms of r, we see that

$$\begin{aligned} \tau(M) &= \frac{2 - r}{2 + r} = \frac{2 - \frac{2}{M}}{2 + \frac{2}{M}} = \frac{M - 1}{M + 1} \\ p_m(M) &= \frac{(M + 1)(M + 2)}{2(M - 1)} = \frac{(1 + r)(2 + r)}{r(2 - r)} \end{aligned}$$

As can be seen, the need for money to make transactions introduces an inefficiency into the sytem. The efficient trade level is $\tau = 1$, but in the monetary equilibrium $\tau(M)$ is always less than 1. (Indeed for $M < 1$ there are not enough gains to trade at e to support any monetary equilibrium.)

As M increases, the nominal rate of interest charged by the bank, $r = (m^1 + m^2)/M = 2/M$, decreases, and the level of trade $\tau(M) \to 1$ as $M \to \infty$.

Unfortunately, as M increases, the price level $p_m(M) = (M+1)(M+2)/2(M-1)$ also increases. If the central bank does not want to allow too high a price level, it will need to keep M low.

For example, if $M = 20$, then $r = (1 + 1)/20 = 10\%$, $\tau = 19/21 \approx 0.90$ and $p_m = 21(22)/38 \approx 12.16$. If $M = 20 + \sqrt{20^2 + 2(20)} = 40.98$, then $r \approx 4.9\%$, $\tau \approx .95$, and $p_m \approx 22.56$. The increase in efficiency from $\tau = .90$ to $\tau = .95$ has been bought by almost a 90% inflation of prices.

The model thus incorporates a simple trade-off between efficiency and the price level, which we shall use as a proxy for the very important trade-off the Federal

Reserve faces between employment (or output) and inflation. In a non-symmetric economy, agents with relatively high cash endowments m^h would be opposed to policy that reduced interest rates and increased trading efficiency, because the higher prices would diminish the value of their cash endowments.

4.2 Credit Card Equilibrium

Suppose credit cards are suddenly introduced. They improve the transactions technology in our model by enabling an extra period of cash flow between commodity trade and bank repayment. This represents the overnight electronic transfer of credit card accounts in the real world. Now the same paper money can do more work, and inevitably price levels rise.

The transactions technology improvement will reduce the wedge between buying and selling, and thus tend to improve welfare. On the other hand, the price level increase will reduce the purchasing power of the each household's stock of outside money m^h, which tends to reduce welfare. When there is no default on credit cards, the benefits from superior technology outweigh the costs of higher prices. But as we shall see, when credit cards default, some of the benefits are frittered away, and welfare tends to go down on account of the inflation.

To illustrate the case without default, we compute the credit card equilibrium for our example, concentrating on the case $M = 20, m^1 = m^2 = 1$.

The introduction of credit cards does not change the rate of interest at the bank. But let us denote the "wedge" between buying and selling by $1 + r^* \equiv \sqrt{1 + \frac{m^1+m^2}{M}}$, so, $1 + r^* < (1 + r^*)^2 = 1 + r$, where r is the bank rate of interest (both before, and after, the introduction of credit cards). For $M = 20$, $r = 10\%$ and $r^* \approx 4.9\%$.

Trade will now be more efficient

$$\begin{aligned} \frac{3-\tau^*}{1+\tau^*} &= 1 + r^* \\ \tau^* &= \frac{2-r^*}{2+r^*} \approx 0.95 > 0.90 \approx \frac{2-r}{2+r} = \tau \end{aligned}$$

But the cash prices nearly double, and the credit card prices are even higher:

$$\begin{aligned} p_m^* &= \frac{(1+r^*)(2+r^*)}{r^*(2-r^*)} = 22.56 > 12.16 = \frac{(1+r)(2+r)}{r(2-r)} = p_m \\ p_c^* &= (1+r^*)p_m^* = \frac{(1+r^*)^2(2+r^*)}{r^*(2-r^*)} = 23.66 \end{aligned}$$

These trades and cash prices are exactly the same as were obtained without credit cards in the pure monetary economy with inside money $M = 40.98$, as stated in Theorem 4. Now we verify that there are indeed corresponding credit card equilibrium actions.

Let each agent h borrow $M/2 = \$10$ from the bank and spend $\$11 = \$1 + \$10$ on the cash market for good $-h$, and thus buying $11/p_m^* \approx 0.49$ of good $-h$ via cash. Let h also spend \$11 on the credit card market for good $-h$ thus buying $11/p_c^* \approx 0.46$ of good $-h$ via the credit card. Finally, let each agent h sell 0.49 units of good h against cash, and 0.46 units of good h against the credit card promise.[10]

Notice that all markets clear. Also, each agent h is able to repay his credit card debt of \$11 from the revenue he obtains from his cash sales. Furthermore, since each agent h borrows \$10 from the bank at interest rate 10%, he must repay \$11 to the bank. But this is precisely what he is paid for his sales against the credit card just before he must go to the bank. The cash flows mirror what is stated in Theorem 3.

These choices are not only consistent, but also optimal for each household h. By selling a unit of good h for cash at price p_m^* and buying $1/(1 + r^*)$ units of good $-h$ via a credit card at price $p_c^* = (1+r^*)p_m^*$ (repaying the credit card debt later with the cash receipt) h faces a wedge of $(1 + r^*)$ between buying and selling. This is also the case if he borrows $p_m^*/(1 + r^*)$ from the bank at interest rate r, then uses the money to purchase $1/(1 + r^*)$ units of good $-h$ for cash at price p_m^*, while simultaneously selling one unit of good h against the credit card at price $p_c^* = (1 + r^*)p_m^*$. The money obtained from the credit card sale will be just enough to repay the bank debt $(1 + r)p_m^*/(1 + r^*) = (1 + r^*)p_m^*$.

Optimality requires that the ratio of marginal utilities is equal to the wedge $(1 + r^*)$, i.e.

$$\frac{x_h^h}{x_{-h}^h} = (1 + r^*)$$

This is exactly how we derived the formula for trades τ^*.

As stated in Corollary 1 to Theorem 4, the introduction of credit cards has created efficiency gains in trade, but at the cost of much higher price levels. Unfortunately, the only way to reduce prices to their old levels is to give back all of the gains in trade, and even a little more!

To get back to the old efficiency levels of trade means an effective interest rate of $r = 10\%$ again. But as we just saw, in the presence of credit cards, that requires a higher bank interest rate of

$$1 + \hat{r} = (1 + r)^2 = 1.1^2 = 1.21$$

which in turn implies

$$\hat{M} = \frac{2}{\hat{r}} = \frac{2}{0.21} = 9.5,$$

a drastic reduction in the money supply.

[10] Thinking of each type as many identical agents, we are here describing a type-symmetric equilibrium in actions. But, as was said after Corollary 1 to Theorem 4, this can be recast in a more realistic manner: some of each type could buy exclusively via credit cards and the rest exclusively via cash without distrurbing the equilibrium allocation.

The price levels will then be

$$\begin{aligned}\hat{p}_m &= \frac{(1+r)(2+r)}{r(2-r)} = 12.16 = p_m \\ \hat{p}_c &= (1+r)\hat{p}_m = 13.38\end{aligned}$$

While the cash prices are restored to their old levels, the credit card prices are 10% higher. Since half the expenditures are by credit card, there is an overall inflation of about 5%. The only way to get rid of this inflation is to reduce the bank supply even more. But this will cut trading efficiency below the levels that prevailed prior to the introduction of credit cards. This illustrates Theorem 5.

It is easy to imagine that the monetary authority might be reluctant to cut money supply so far. It might well stop at a point where average trades are just a tad below 0.90 and prices are a tad above 12.16. This is a touch of stagflation.

5 Default and Stagflation

Credit card purchases conjure the possibility of *default*, so we take the opportunity to introduce default into our models. Households will no longer be obliged to repay their credit card debts; they may default.

If credit cards default, we must allow lenders to foresee this, and to charge a "default premium", raising the credit card prices further above the cash prices. We suppose for simplicity that a single credit card is usable across all goods. All credit card cash payments are aggregated (across agents and commodities), and the ratio of these payments to the aggregate credit card promises determines an average delivery rate K for each merchant who sold commodities against credit cards. The shortfall $1-K$ is the default rate on credit cards, and from the merchant's point of view can be interpreted as a proportional fee on his credit card sales.

Deliveries made by "unreliable" households and "reliable" households are pooled. As in real life, there is no practical device to easily separate them. Hence the reliable credit card purchasers pay for the defaults of the unreliable purchasers via the higher credit card prices.[11]

To introduce default into our model we will first extend our previous example by thinking of two types of agents: reliable Dr. Jekylls whose characteristics are exactly as before and who keep all their credit card promises, and unreliable Mr. Hydes who always default. For simplicity, we suppose that Hyde always charges $\tau_h = .03$ units

[11] One might even say that paying the default penalty allows the defaulters to steal from the reliable purchasers. In the multiperiod world in which we live things are even worse. Credit card prices and cash prices are generally the same, so credit card purchasers who pay on time are effectively getting a month's free interest. Reliable puchasers who pay late not only have to pay interest (sometimes as much as 18% per annum) in order to make up for the defaulters, but also to make up for the interest free loans given to the automatic payers. These subtleties are necessarily absent in our one-period model.

of each good on his credit card, eats the goods he thereby purchases, and then simply defaults fully on his promises, and that he does nothing else. In effect the Hydes are simply stealing $\tau_h = .03$ from the Jekylls.

Our first conclusion will be that despite the sudden increase in demand from the Hydes, the volume of trade is reduced by credit card default. As K falls the wedge between credit card prices and cash prices rises, inhibiting trade. But even with default on credit cards, the volume of trade can never be lower than it would be were credit cards eliminated altogether.

Our second conclusion is that default accentuates the inflation brought on by credit cards. This is so for two reasons. First, the bigger wedge caused by default reduces total sales of commodities, as we already said. Second, somewhat strangely, default raises the proportion of goods sold against credit cards. The two reasons work together to raise p_m because the same money $M + \bar{m}$ is chasing fewer commodities on the cash market.

Our third conclusion is that the utility to the Jekylls is lower than if there never were any credit cards. This is quite surprising. The Jekylls could choose to transact entirely in the cash markets, where there is no chance of default or scam. If they all did so, then the previous cash-only equilibrium would be reached. But the convenience of credit cards makes them irresistable if defaults are not too high. Once many Jekylls buy and sell via credit cards, the volume of goods sold against cash is lowered, raising the cash prices p_m, as we just observed. That reduces the purchasing power of each Jekyll's endowment of money. Thus no Jekyll by himself can replicate his trade in the cash-only equilibrium.

Thus credit cards can be bad for the economy when default occurs. They can create a gigantic inflation and at the same time reduce the utility of honest Jekylls. If the monetary authority, alarmed by this inflation, tries to correct the situation by reducing the money supply M, it will make things worse. The volume of trade will fall, as will utilities. At the point where price levels are restored to their pre-credit card levels, the volume of trade will be lower, and the utilities of the Jekylls will be much worse. The monetary authority might very well stop short of reducing M so far, leaving a new equilibrium with higher prices, less volume of trade, and lower utilities. Thus with default on credit cards, we get not just a touch of stagflation, but robust stagflation, as we shall see in the next Sections.

5.1 An Example of Robust Stagflation with Constant Default

In doing comparative statics on an economy with credit card defaults, we need to be explicit about how default will change as the economy grows poorer or richer. One might guess, for instance, that as the economy grows poorer, agents will cheat more out of desperation. In our first example we will keep things simple, supposing that the amount of credit card fraud τ_h by the Hydes remains constant, even as we change parameters such as the quantity of bank money M.

Suppose we are at an equilibrium in which the Jekylls are actually buying on the credit card markets (as well as on cash markets). Letting τ_m and τ_c be the quantities sold by the Jekylls against cash and against credit cards, respectively, we must have

$$\begin{aligned} K &= \frac{\tau_c - \tau_h}{\tau_c} \\ \frac{M + \bar{m}}{2\tau_m} &= p_m \\ p_m \tau_m &= p_c(\tau_c - \tau_h) \\ \frac{(1+r)p_m}{Kp_c} &= \frac{p_c}{p_m} \\ \frac{(3 - \tau_m - \tau_c)}{(1 + \tau_m + \tau_c - \tau_h)} &= \sqrt{\frac{(1+r)}{K}} \\ r &= \frac{\bar{m}}{M} \end{aligned}$$

Jekyll's purchase on each credit card market is $\tau_c - \tau_h$ since his purchase plus Hyde's purchase of τ_h must equal the total supply of τ_c. Since he delivers fully and Hyde does not deliver at all, the rate of delivery on credit card promises is given by the first equation. The second equation comes from the fact that all the money $M + \bar{m}$ is spent in the two cash markets, and that Jekylls are selling τ_m there, while Hydes do not participate on the cash markets. The third comes from the fact that each Jekyll uses all his cash receipts to pay off all his credit card debt.

The fourth equation (which may also be written $p_c/p_m = \sqrt{(1+r)/K}$) equates the two strategies of trading in a CCE for an agent like Dr. Jekyll who is not defaulting. On the left hand side is the quantity of good that must be sold on the credit card market in order to repay the bank loan required to buy an incremental unit on the cash market. On the right hand side is the quantity of good that must be sold on the cash market in order to repay the debt incurred buying an incremental unit on the credit card market. Since Jekyll is buying on both markets, these must be equal.

The fifth equation says that the wedge between buying with borrowed cash and selling on credit cards (or vice versa), which is $\sqrt{(1+r)/K}$, as we just saw, must equal the ratio of Jekyll's marginal utility of buying and selling. The sixth equation holds as before because there is no bank default.

Solving these equations for $M = 20, 16, 12, 9$ gives the last four columns of the table below:

The first numerical column recalls the pure monetary equilibrium without credit cards from Section 4.1, and the second column recalls the credit card equilibrium (with no default, i.e. with Hyde absent and $\tau_h = 0$) from Section 4.2. The third column describes the CCE with credit card default (Hyde's defaults $\tau_h = .03$.) The

M	20	20	20	16	12	9
th	0	0	0.03	0.03	0.03	0.03
tm	0.905	0.487597	0.470613	0.467141	0.461369	0.453699
pm	12.16	22.5596	23.37379	19.26612	15.17224	12.12257
tc	0	0.464901	0.463961	0.455679	0.442407	0.425661
pc		23.6609	25.34788	21.14269	16.97349	13.90079
K		1	0.935339	0.934164	0.932189	0.929521
tm+tc-th	0.905	0.952498	0.904574	0.89282	0.873776	0.84936
tm+tc	0.905	0.952498	0.934574	0.92282	0.903776	0.87936
J-utility	1.384035562	1.38573	1.369595	1.369079	1.368093	1.366558

Figure 1:

remaining columns maintain this credit card default at a constant level, and examine the effect on CCE of the monetary authority lowering M.

Let us begin by moving from column one to two to three to verify our first three conclusions.

Observe first that the appearance of Hyde into a world with credit cards engenders default and reduces total trade $\tau_m + \tau_c$ from .95 to .935., but maintains it higher than the .905 it was without credit cards. The wedge between buying and selling is $1+r$ in the cash-only economy, is reduced to $\sqrt{1+r}$ by credit cards, but rises to $\sqrt{(1+r)/K}$ when credit card default occurs. This explains why the volume of trade rises with credit cards and then falls with default.

The appearance of Hydes, who commit credit card default, is bad for all the honest Jekylls, though of course the Hydes benefit from their "thefts". Jekyll's utility rises slightly from 1.384 without credit cards to 1.386 with credit cards, and then falls all the way down to 1.370 with credit card default. The reason is that even without credit cards, the pure monetary equilibrium was close to Pareto efficient. The introduction of credit cards substantially raised the volume of trade, but since most of these trades were only minor welfare improvements, it had only a second order effect on increasing the utility of the Jekylls. The introduction of credit card default reduces the volume of trade, which again has a minor effect on utility, but it allows the Hydes to steal from the Jekylls, and this has a first order effect on the Jekylls' utilities. Thus the loss from thefts outweights the gains from the extra trades made via credit cards, explaining why Jekyll has lost utility in moving from column one to column three.

Price levels shoot up from 12.16 without credit cards, to 22.56 with credit cards, and shoot up even more to 23.37 with credit card default. Thus credit card default accentuates the inflation. There are two reasons. First, the total volume of trade is reduced when credit card default sets in, as we have seen. Second, the fraction $\tau_m/(\tau_m + \tau_c)$ of trade on cash markets falls from $.4876/.9525 = 51.2\%$ to $.4706./.9346 = 50.4\%$ as credit card defaults rise. With the same money $M + \bar{m}$ chasing fewer goods on the cash markets, prices must rise.

The shift from cash goods to credit goods, even as credit card deliveries become less reliable, seems paradoxical. The explanation depends on general equilibrium: prices and quantities must adjust so that the Jekylls are still indifferent between the buy-cash/sell-credit and the buy-credit/sell-cash strategies (equation 4 above), and at the same time the Jekylls must obtain just enough cash revenue to pay off all their credit card debt (equations 1 and 3 above). Equations 1,3, and 4 together give $\tau_m/\tau_c = \sqrt{K(1+r)}$.[12]

Since credit card prices are even higher than the cash prices, we see that the introduction of credit cards with default has created an inflation of nearly 100% from column one to column three.

An alarmed monetary authority is likely to react by cutting the money supply M. But as we see in columns three through six, this action does not simply undo the inflation. As prices fall, interest rates $r = 2/M$ rise and total trade $\tau_m + \tau_c$ declines. By the time $M = 12$, trade has already fallen to .904, just below its pre-credit card level of .905. But cash price levels are still grossly inflated at $p_m = 15.17$, twenty percent above the pre-credit card levels of 12.16, and credit card prices are of course higher still at 16.97, giving an average price increase of 24%. If the monetary authority tries to lower prices any further, it will create stagflation, lowering total trade while prices remain above pre-credit card levels. The stagflation persists all the way down to $M = 9$, by which time prices are back to their pre-credit card levels, but trade is down two and a half percent from .905 to .879. Of course utility for the Jekylls is dramatically lower at 13.67. Unlike the situation of Section 4.2, where credit cards created only a touch of stagflation, here they create a robust stagflation on account of default.

6 Credit Card Default: A General Model with Nominal Penalties

We now present a general model of credit card default with nominal penalties, in which default levels and default rates are determined endogenously in equilibrium. Households are not obliged to repay their credit card debts, but if they fail they

[12] The appearance of Hydes obviously lowers K. One might mistakenly guess that p_c would rise enough to maintain equality between Kp_c and p_m. But that would be based on the erroneous supposition that agents are indifferent between selling against credit cards and against cash. They are not, because they get the money sooner if they sell against cash. If p_c were to rise so high, then the buy-cash/sell-credit cards strategy would be just as good as before. But with such a big rise in p_c, the buy-credit/sell-cash strategy would be much worse, contradicting the needed indifference of Jeckyll between the two strategies. Instead, prices must adjust so that both strategies decrease in attractiveness together, which can only happen if the buy-cash/sell-credit strategy also gets worse and p_m/Kp_c rises. Since total cash receipts $p_m\tau_m$ must continue to be equal to total credit card deliveries $Kp_c\tau_c$, τ_m/τ_c must fall, as claimed.

must suffer a penalty modeled as a disutility rate λ per *dollar* value of default.[13] If the default penalties are high enough, then the model reduces to the preceding case where default is ruled out by assumption. If the default penalties are too low, then nobody will repay their credit card debt, and credit cards will cease to be used. But for intermediate levels of penalties, credit cards will be used even though there is default. (Having allowed for default on credit cards, we also allow for default on bank loans, with a corresponding penalty rate $\tilde{\lambda}$.)

Our first conclusion is that credit card equilibrium exists for all $\lambda > 0$, under the same gains to trade hypothesis $\gamma(e) > \bar{m}/M$ as was postulated in Theorem 1 for the model without credit cards.[14] Thus credit cards do not endanger the viability of money, no matter how low the default penalties.

Next we show via an example that the conclusions of the previous section continue to hold with nominal default penalties. In particular, credit card default leads to a big stagflation if the monetary authority tries to intervene to eliminate the inflation brought on by credit cards.

We also get a new phenomenon, namely the catastrophic abandonment of credit cards if the monetary authority reduces M too far. In our previous example, total defaults were assumed to be independent of M. Now we find that if M is reduced, prices fall, so defaulting by a dollar enables one to buy more goods while suffering the same old penalty λ. Default rates rise rapidly and eventually honest people are led to shun credit cards altogether.

6.1 Notation

We permit, but do not require, *credit limits* on cardholders, which presumably reflect their credit history and estimates of their income/wealth. This bounds the damage unreliable types can do to the average delivery rate. We model these limits as exogenous.

At the end of the section we vary the model to allow for netting (though at the present time this is not observed in practice). In order for credit card equilibrium to exist with netting, it now becomes necessary to require that that there are households with finite credit limits, or else that there are some "cash-only" commodities in the economy which cannot be purchased with credit cards.[15] Were *all* commodities liable to *unlimited* credit card purchase, money would lose value with netting, and ME would cease to exist.

Let us turn to a precise formulation of the model. Let

L_1 = the set of cash-only commodities (L_1 could be empty).

[13] In a one-period model, there is no room for more sophisticated penalties in terms of restrictions on future economic activity.

[14] The viability of money relies on our assumption that as prices go to infinity, the default penalty per dollar stays bounded away from 0.

[15] Think of cab rides in Manhattan or street vendors in Mumbai.

$L_2 = L \backslash L_1 =$ the set of commodities accessible to credit cards, as well as money. For $\ell \in L_2$, let

$p_{\ell c} \equiv$ credit card price of ℓ

and, for $\ell \in L$, let

$p_{\ell m} \equiv$ cash price of ℓ.

Thus market prices are now given by a vector $(p, r) \equiv ((p_{\ell m})_{\ell \in L}, (p_{\ell c})_{\ell \in L_2}, r)$ with $L + L_2 + 1$ components. In addition to (p, r), an ME will now have another "macrovariable" by way of a scalar $0 \leq K \leq 1$, where

$K \equiv$ the expected rate of delivery on the credit card.

Before describing the budget-set we introduce $(C^h, \lambda^h, \tilde{\lambda}^h) \in \overline{\mathbb{R}}_+ \times \overline{\mathbb{R}}_{++} \times \overline{\mathbb{R}}_{++}$ for $h \in H$, where $\overline{\mathbb{R}}_+ \equiv \mathbb{R}_+ \cup \{\infty\}$, $\overline{\mathbb{R}}_{++} \equiv \mathbb{R}_{++} \cup \{\infty\}$ and

$C^h \equiv h$'s spending limit on the credit card

$\lambda^h \equiv h$'s penalty rate for default on credit card loans

$\tilde{\lambda}^h \equiv h$'s penalty rate for default on bank loans.

(For simplicity, all three are scalars. We could in general let them be continuous functions which depend on (p, r, K), the volume of trade, and other macrovariables of the economy. This would leave our analysis intact.)

We denote the credit card economy by $(\mathcal{E}, C, \lambda, \tilde{\lambda}, m, M, L_1)$.

Note that the penalty rates $(\lambda^h, \tilde{\lambda}^h)$ may or may not be infinite, but must be strictly positive. Also note that we allow $L_1 = \emptyset$ and $C^h = \infty$. When we introduce netting, we will be forced to require either $L_1 \neq \emptyset$ or some $C^h < \infty$.

6.2 Budget Set without Netting

The vector q^h of market actions of household h now has[16] $2L + 2L_2 + 1$ components, with

$q^h_{\alpha\beta} \equiv$ quantity of α sent by h to the market $\alpha\beta$

where the markets are nm (the bank loan market), $(\ell c)_{\ell \in L_2}$ (the credit card-commodity markets), $(\ell m)_{\ell \in L}$ (the cash-commodity markets). In addition to q^h, household h needs also to choose

$\delta^h \equiv$ money repaid on credit card

$\tilde{\delta}^h \equiv$ money repaid on bank loan.

Given (p, r, K), the vector $(q^h, \delta^h, \tilde{\delta}^h, x^h)$ must satisfy:

$$\tilde{m}^h \equiv \frac{q^h_{nm}}{1 + r} \tag{1'}$$

$$\sum_{\ell \in L} q^h_{m\ell} \leq m^h + \tilde{m}^h \tag{2'}$$

[16]Since bank deposits are returned after clearing debts on the credit card, once again no household will need to deposit money at the bank, and we suppress deposits.

$$\sum_{\ell \in L_2} q^h_{c\ell} \leq C^h \tag{3'}$$

$$q^h_{\ell c} + q^h_{\ell m} \leq e^h_\ell, \text{ with } q^h_{\ell c} \equiv 0 \text{ for } \ell \in L_1 \tag{4$'\ell$}$$

$$\tilde{x}^h_\ell(m) \equiv \frac{q^h_{m\ell}}{p_{\ell m}}, \; \tilde{x}^h_\ell(c) \equiv \frac{q^h_{c\ell}}{p_{\ell c}}, \text{ with } q^h_{c\ell} \equiv 0 \text{ for } \ell \in L_1 \tag{5$'\ell$}$$

$$\delta^h \leq \Delta(2') + \sum_{\ell \in L} p_{\ell m} q^h_{\ell m} \tag{6'}$$

$$\tilde{\delta}^h \leq \Delta(6') + \sum_{\ell \in L_2} K p_{\ell c} q^h_{\ell c} \tag{7'}$$

$$x^h_\ell \leq \Delta(4'\ell) + \tilde{x}^h_\ell(m) + \tilde{x}^h_\ell(c) \tag{8$'\ell$}$$

The budget set $B^h(p, r, K)$ of household h consists of all $(q^h, \delta^h, \tilde{\delta}^h, x^h)$ that satisfy constraints $(1')$ to $(8')$. These conditions are exactly as before, except for $(3')$, which describes the credit limit, and $(6')$ and $(7')$ which say that deliveries must be made out of the money on hand and that credit card deliveries are due before bank loans.

6.3 Payoffs

Household h has promised to pay $\sum_{\ell \in L_2} q^h_{c\ell}$ and q^h_{nm} on his credit card and bank loan respectively. Therefore his total payoff, including default penalties, is

$$\Pi^h(q^h, \delta^h, \tilde{\delta}^h, x^h) = u^h(x^h) - \lambda^h \max\left\{0, \sum_{\ell \in L_2} q^h_{c\ell} - \delta^h\right\} - \tilde{\lambda}^h \max\{0, q^h_{nm} - \tilde{\delta}^h\}.$$

6.4 Credit Card Equilibrium without Netting

Unrefined Credit Card Equilibrium We say that $\langle p, r, K, (q^h, \delta^h, \tilde{\delta}^h, x^h)_{h \in H} \rangle$ is an *unrefined CCE* of the economy $((u^h, e^h, m^h, C^h, \lambda^h, \tilde{\lambda}^h)_{h \in H}, M, L_1)$ if:

(i) $(q^h, \delta^h, \tilde{\delta}^h, x^h) \in B^h(p, r, K)$ and $\Pi^h(q^h, \delta^h, \tilde{\delta}^h, x^h) \geq \Pi^h(\underline{q}^h, \underline{\delta}^h, \underline{\tilde{\delta}}^h, \underline{x}^h)$ for all $(\underline{q}^h, \underline{\delta}^h, \underline{\tilde{\delta}}^h, \underline{x}^h) \in B^h(p, r, K)$

i.e., all agents optimize on their budget sets and

(ii) $\sum_{h \in H} \tilde{m}^h = M$

$\sum_{h \in H} \tilde{x}^h_\ell(m) = \sum_{h \in H} q^h_{\ell m}$ for $\ell \in L$

$\sum_{h \in H} \tilde{x}^h_\ell(c) = \sum_{h \in H} q^h_{\ell c}$ for $\ell \in L_2 \equiv L \backslash L_1$

i.e., all the $L_2 + L + 1$ markets clear and

$$\text{(iii)}\ \ K \equiv \begin{cases} \sum_{h\in H} \delta^h / \sum_{h\in H} \sum_{\ell\in L_2} q^h_{c\ell} & \text{if the denominator is positive} \\ \text{arbitrary,} & \text{otherwise.} \end{cases}$$

In equilibrium a seller charges $p_{\ell c}$ to any customer who wishes to purchase commodity ℓ with the credit card. The seller only expects to receive $Kp_{\ell c}$ from the credit card company. In fact $(1-K)$ is the average rate of default of credit card users. But the seller need not know this. To him the $(1-K)$ appears as a "user tax" levied by the credit card company, which guarantees the seller full delivery (net of the tax) on all credit card purchases. In this interpretation of equilibrium, competition between potential credit card companies would drive their profits to zero, forcing the equality of the user tax and the endogenous default rate $1-K$.

As it stands there is a problem with our definition. Nothing prevents the expectation K from being absurdly pessimistic when there is no activity on the credit cards (since we allow K to be arbitrary in this case). But then this pessimism in turn ensures that the inactivity will be justified.

Indeed consider an ME without the credit card markets, with prices $(p_{\ell m})_{\ell\in L}$. Now set the prices $(p_{\ell c})_{\ell\in L_2}$ much higher. No household will wish to make credit card purchases because then he would be obliged to clear his debts on the credit card by selling relatively huge amounts of other goods at the prices $(p_{\ell m})_{\ell\in L}$ in the cash-commodity markets; or else undertake to pay a large penalty according to λ^h. Hence the demand will be zero on the $\{\ell c\}_{\ell\in L_2}$ markets. Now suppose that K is set (positive but) with $K \max_{\ell\in L_2}\{p_{\ell c}\}$ close to zero. Then in spite of the high prices $p_{\ell c}$, no household will wish to sell on the $\{\ell c\}_{\ell\in L_2}$ markets because he expects almost no money in return. So the supply will also be zero, and we will have created an unrefined CCE with no activity on the credit cards. To eliminate these spurious equilibria, we introduce an equilibrium refinement.

6.4.1 An Equilibrium Refinement

We rule out arbitrary inactivity on the credit card market by adding one more condition to our definition of equilibrium. An unrefined CCE will be called a CCE if: for all small $\varepsilon > 0$, there exists an "active, optimistic ε-equilibrium" which is "ε-close" to the unrefined CCE. An "optimistic ε-equilibrium" is an equilibrium in which all agents optimize as before, but where we add an external dummy, who puts up $0 < \varepsilon_{\alpha\beta} \leq \varepsilon$ of α on every $\alpha\beta$ market and who fully redeems all his debts. It is "active" if for every commodity $\ell \in L_2$, there is an agent who sells positive quantities of ℓ against credit cards.

Note that the dummy never defaults, so if $K < 1$, there must be real agents making positive credit card purchases along the sequence at every ε. However, in the limit at the CCE itself, it may well happen that there is no active trade in the credit card markets even though credit cards are priced via p_c and K.

6.5 Existence of CCE

For simplicity we shall from now on maintain the assumption that utilities are smooth. Let Δ be the least upper bound on marginal rates of substitution of commodities across all individually rational allocations, i.e., letting $\mathcal{A} = \{x \equiv (x^h)_{h \in H} \in \mathbb{R}_+^{LH} : \sum_{h \in H} x^h = \sum_{h \in H} e^h,\ u^h(x^h) \geq u^h(e^h)$ for all $h \in H\}$, we have

$$\Delta = \sup_{x \in \mathcal{A}} \left\{ \frac{\nabla_\ell^h(x^h)}{\nabla_k^h(x^h)} : h \in H;\ \ell, k \in L;\ x \equiv (x^h)_{h \in H} \in \mathcal{A} \right\}$$

where $\nabla_\ell^h(x^h) = \frac{\partial u^h(x^h)}{\partial x_\ell}$. *Throughout Section 6 we assume:*

$$\Delta < \infty.$$

Theorem 6 *Let $(\mathcal{E}, C, \lambda, \tilde{\lambda}, m, M)$ be a credit card economy with $\gamma(e) > \bar{m}/M$, and (recall) with $0 < \lambda, \tilde{\lambda} \leq \infty$. Then a CCE exists.*

6.6 Efficiency of Credit Card Equilibrium with Default

It should be no surprise that default reduces the efficiency of trade. If $K < 1$, because some unreliable party is buying and then defaulting on his credit card, a seller will require a higher price to be induced to give up his goods. A reliable buyer who has every intention of repaying his credit card will then be discouraged from purchasing. He must pay more than the seller receives, and this wedge between buying and selling prices accounts for the reduction in trading efficiency.

If the reliable buyer himself thinks to make life easier by partially defaulting, the inefficiency argument is unchanged. As long as he delivers something, we can treat his marginal action as if he fully delivered, and we are back to where we were before. It is as if he is a split personality: the reliable Dr. Jekyll fully delivers while the unreliable Mr Hyde fully defaults. The following theorem relies on the hypothesis that everyone delivers at least something on his credit card debts.

We define a CCE to be *interior* if every agent makes positive deliveries on both his credit card and bank loans, and furthermore, buys (or sells) a commodity via cash if and only if he also buys (or sells) the same commodity via credit card. We also require that credit limits are not binding. In symbols, we must have for all $h \in H$

$$\begin{aligned} x^h &> > 0 \\ \delta^h &> 0,\ \tilde{\delta}^h > 0 \\ q_{m\ell}^h &> 0 \Longleftrightarrow q_{c\ell}^h > 0 \\ q_{\ell m}^h &> 0 \Longleftrightarrow q_{\ell c}^h > 0 \\ \sum_{\ell \in L} q_{c\ell}^h &< C^h \end{aligned}$$

Theorem 7: *Let $\langle p, r, K, (q^h, \delta^h, \tilde{\delta}^h, x^h)_{h\in H}\rangle$ be an interior CCE of a credit card economy $(\mathcal{E}, C, \lambda, \tilde{\lambda}, m, M, L_1 = \emptyset)$ without netting.* Then

$$1 + \gamma((x^h)_{h\in H}) = \sqrt{\frac{1 + \frac{\bar{m}}{M}}{K\tilde{K}}} = \sqrt{\frac{1+r}{K}} \leq 1 + r$$

where

$$\tilde{K} = \frac{\sum_{h\in H} \tilde{\delta}^h}{\sum_{h\in H} q^h_{nm}}$$

denotes the average delivery rate on bank loans.

According to Theorem 7, as long as equilibrium is interior, lowering default penalties λ and $\tilde{\lambda}$ will lower delivery rates K and $\tilde{K}$, and thus increase the gains to trade $\gamma((x^h)_{h\in H})$ left at the final allocation. The efficiency of trade declines.

But there is a limit of r on the inefficiency. In general r is endogenous but, if the $\tilde{\lambda}^h$ are all sufficiently high, there will be no default on bank loans, so that $\tilde{K} = 1$ and $r = \bar{m}/M$, and the gains to trade $\gamma((x^h)_{h\in H})$ left at the CCE will be at most $\bar{m}/M$. Thus even with default, the efficiency of trade with credit cards must be at least as good as the efficiency of trade without credit cards.

The formula for $\gamma((x^h)_{h\in H})$ given by Theorem 7 implies that efficiency is steadily deteriorating as K falls. Since there is a limit on the fall, there has to come a catastrophic point at which households will abruptly cease using credit cards and there will be no interior CCE.

6.7 Stagflation with Credit Card Default

Suppose that the bank penalties $\tilde{\lambda}^h$ are fixed and high enough to eliminate bank default. As the credit card penalties λ^h are lowered from infinity, credit card default will eventually set in. We illustrate, in the following example, the main effects discussed earlier. As λ falls: (i) defaults rise; (ii) the total quantity of trade falls; (iii) the fraction of trade conducted via credit cards rises; (iv) price levels rise; (v) stagflation emerges; (vi) a threshhold λ^* is reached below which credit cards are abruptly abandoned and the economy reverts to a pre-credit card, pure cash monetary equilibrium.

6.7.1 An Example

Consider our example of Section 4.2. Now, however, we allow for default on credit cards by taking λ sufficiently small, but we maintain the assumption that nobody defaults on bank loans by setting $\tilde{\lambda}$ sufficiently large.

Symmetry considerations will again allow us to quickly compute equilibrium. Let τ_m (τ_c) be the quantity of each good traded on the cash market (credit card market).

We look for interior CCE with default on credit cards. The following conditions must hold:

$$\begin{aligned}
\frac{M+\bar{m}}{2\tau_m} &= p_m \\
\frac{p_m\tau_m}{p_c\tau_c} &= K \\
\frac{1}{(1+\tau_m+\tau_c)} &= \lambda p_c \\
\frac{1}{(3-\tau_m-\tau_c)} &= \lambda p_m \\
\frac{(1+r)p_m}{Kp_c} &= \frac{p_c}{p_m} \\
r &= \frac{\bar{m}}{M}
\end{aligned}$$

The first equation comes from the fact that all the money is spent in the cash markets. The second comes from the fact that all the money a household obtains from sales in the cash market is used to pay off his credit card dues. The third equation says that a household is indifferent between buying another unit via the credit card and defaulting completely on this incremental purchase. The fourth says that he is indifferent to selling an incremental unit in the cash market and using the proceeds to reduce his credit card default.

The fifth equation is as before, and makes the Dr. Jekyll side of each agent indifferent between the buy-cash/sell-credit and buy-credit/sell-cash strategies. The sixth equation holds because there is no bank default.

With $M = 20$ and $m^h = 1$ (so $\bar{m} = 2$), we have $r = 2/20 = 0.1$. With $\lambda \geq 0.02165$ we get the no-default equilibrium of section 4.2. Below we give a short table of equilibrium values as λ falls below 0.0216. Notice that there is no credit card default in the first numerical column, and that credit cards are not used in the last two columns. Indeed, credit cards will only be used when the "credit card wedge" $\sqrt{(1+r)/K} = \sqrt{1.1/K}$ between buying and selling does not exceed the "cash wedge" $1+r = 1.1$.

M......	20.000	20.00	20.00	20.00	20.000	20.0000	20.00
λ	.02165	.0210	.0205	.02	.01965	.01965	.01
K	1	.971	.948	.926	.910	.455	.232
$\sqrt{1.1/K}$	1.049	1.065	1.077	1.09	1.1	1.55	2.18
p_m	22.56	23.09	23.52	23.97	24.30	12.16	12.16
p_c	23.66	24.58	25.33	26.12	26.71	26.72	52.50
τ_m	.488	.476	.468	.459	.453	.905	.905
τ_c	.465	.461	.458	.455	.452	0	0
$\tau_m+\tau_c$	.952	.937	.926	.914	.905	.905	.905
netutility	1.386	1.378	1.373	1.367	1.3627	1.3840	1.3840

Default Penalty Stagflation Consider the effects of lowering the default penalties from 0.02165, where default was prohibitively painful and so did not occur ($K = 1$), down to 0.01965 at which point $K = .910$.

The drop in default penalties λ causes an increase in default, and thus a decrease in delivery rates K, confirming (i) above. This increases the wedge $p_c/p_m = \sqrt{(1+r)/K}$ and reduces the level of real trade $\tau_m + \tau_c$, confirming (ii).

Next, rearrange the second equation and the fifth equation in Section 6.7.1. We see that

$$\frac{\tau_c}{\tau_m} = \frac{p_m}{Kp_c} = \frac{1}{\sqrt{K(1+r)}}$$

which shows why τ_c/τ_m is rising, confirming (iii).

Cash prices p_m rise as the same total money $M + \bar{m}$ chases fewer goods $2\tau_m$, confirming (iv).

Net utilities fall for three reasons: trade diminishes on account of increases in the wedge; the total default penalty $\lambda(p_c\tau_c - p_m\tau_m)$ increases because default increases more than λ falls; the purchasing power of the initial cash endowments of outside money m^h falls since cash prices p_m rise.

The reduction in default penalty rates λ thus causes a stagflation: prices go up, and output (trade) and utilities go down, confirming (v).

We study (vi) in the next section.

The catastrophic elimination of credit cards Once K falls to .910, it is no longer disadvantageous to trade exclusively on the cash market, as compared to buying with a credit card and selling for cash or vice versa. This is reflected in the fact that the wedge $p_c/p_m = \sqrt{(1+r)/K} = \sqrt{1.1/.910} = 1.1 = 1 + r$ has risen to the gross bank interest rate of 1.1. Any further reduction in the penalty λ will increase the wedge still further and cause households to abandon credit cards altogether, as we see in the last column where $\lambda = .01$ and $t_c = 0$.

At the point where $\lambda = .01965$, there are actually two equilibria, with identical real trade but very different prices (and different utilities). The first (described by the fifth numerical column) involves the use of both cash and credit cards, and the second (shown in the sixth numerical column) is cash only. The first equilibrium is obtained by solving the same equations as in 6.7.1. But in the second equilibrium credit cards have been abandoned in favor of cash. This is so because, with low delivery rates K, credit card prices p_c are so high that credit card purchases become unattractive to the Dr. Jekylls who intend to keep their promises

$$p_c/p_m = 1/K = 1/.455 > \sqrt{(1+r)/K} = \sqrt{1.1/.455} > 1.1 = 1 + r$$

and nobody will take credit from the Mr. Hydes. This is the start of a new regime, of cash-only equilibria. We compute these equilibria from the pure monetary equilibrium (described in Section 4.1) as follows. Set $\tau_c = 0$, then compute p_c from the third equation of 6.7.1, and finally compute K from the equation $Kp_c = p_m$. This last equation holds because, in the trembles of the refinement, sellers must be indifferent to raising a dollar via cash sales and credit card sales. (That was not true before because the proceeds from the cash sales were used entirely to pay off credit card debts before credit card receipts were obtained. In the trembles from cash-only equilibria, most of the cash receipts are kept around to pay the bank loans). As λ falls further below the threshhold .01965, we remain in the cash-only regime, with the same transactions and same cash prices, but ever rising p_c and falling K.

It is noteworthy that at the threshhold $\lambda = .01965$, there is a catastrophic change in regime. Credit cards abruptly disappear (taking with them the default penalties), net utility jumps up, and p_m falls dramatically.

Utilitarian stagflation caused by credit cards Supose we are in a pure monetary equilibrium, as in the example from Section 4.1, and then credit cards are introduced, as in Section 4.2. Suppose, however, that the credit cards do not carry infinite default penalty rates; for concreteness, suppose the penalty rate is $\lambda = .02$.

In the new equilibrium that emerges, cash prices p_m nearly double, from 12.16 to 23.97, while net utility falls from 1.384 to 1.367. There is, to be sure, a slight increase in trade which goes up from .905 to 914. The utility gains from the extra trade are, however, more than offset by the default penalties that households incur.

Recall that there are two negative effects of credit cards on household utilities. The first is that the higher inflation reduces the purchasing power of their money endowments. The second is the deadweight loss of utility when they default. The benefit is a lower wedge between buying and selling. When default penalties are so high as to prevent default, as in the first numerical column, the reduction in the wedge (from $(1 + r) = 1.1$ to $\sqrt{(1 + r)} = 1.049$) is significant enough to offset the disadvantages of credit cards. But with default, the wedge is improved less, from $(1 + r) = 1.1$ to $\sqrt{(1 + r)/K} = 1.09$, and so we see a net loss in welfare in the transition from the pure monetary economy to credit cards. Indeed this is true in all the numerical columns 2 till 5.

Thus the introduction of credit cards with default dramatically raises prices and simultaneously lowers utilities.

Stagflation caused by the reaction of monetary policy to credit cards Suppose now that the monetary authority is alarmed by the nearly 100% increase in price levels, from 12.16 to 23.97, caused by the introduction of credit cards (with penalty rates .02). If the monetary authority reacts to the terrible inflation by gradually shrinking the money supply from $M = 20$ to $M = 19.57$, then we get a new equilibrium shown below in the fourth column. Prices do decline slightly to 23.85, but at the cost of reducing trade to .904.

20	M	20	19.57	19.5
–	λ	.02	.02	.02
–	K	.926	.909	.453
–	$\sqrt{(1+2/M)/K}$	1.09	1.101	1.562
12.16	p_m	23.97	23.85	11.91
–	p_c	26.12	26.26	26.28
.905	τ_m	.459	.452	.902
–	τ_c	.455	.452	0
.905	$\tau_m + \tau_c$	.914	.904	.902
1.3840	$netutility$	1.367	1.362	1.3839

Compared to the pre-credit card, pure monetary equilibrium levels (shown in the far left column), the monetary authority has engineered not merely utilitarian stagflation, but the full-blown stagflation of (much) higher prices (from 12.16 to 23.85) and (slightly lower) trade/output (from .905 to .904).

If the monetary authority tries to further reduce prices by shrinking M still more, it will quickly drive out credit cards altogether. When $M = 19.5$, credit cards have already been driven out, as shown in the last column. At this point prices are restored to slightly less than the pre-credit card level of 12.16, because M is smaller. But output is also smaller.

The upshot is that in order to undo the inflationary effects of credit cards, it is necessary to tighten money supply M so much as to banish credit cards altogether. But this entails reducing output/trade to below its pre-credit card levels, since there is now less money and higher interest rates, and no credit cards in use.

6.8 Real Default Penalties

Penalties so far have been based on the dollar value of the default. They could instead be indexed by a basket of goods, so that in times of high inflation they do not become astronomically harsh. For instance, suppose an agent has additively separable utility $u(x) = \sum_{\ell=1}^{L} u_\ell(x_\ell)$ and suppose the penalty for each dollar's worth of default is λ/p_{Lc}. Then if the agent is defaulting in equilibrium, his consumption of good L is fixed by $\partial u_L(x_L)/\partial x_L = \lambda$. The Hydes in our example of Section 5 could have been generated via real penalties with an appropriately chosen λ, as could the Jekylls for whom we could imagine $\lambda = \infty$.

6.9 Postscript: Credit Card Equilibrium with Netting

It is imaginable that in some future era, electronic accounting will enable credit card accounts to be netted, so that a household that runs a business will be able to subtract its outstanding credit card payables from its own credit card debt. This turns out to make equilibrium more problematic, but we show it still does exist if there are credit card limits or cash-only commodities. Here we outline the details.

The budget set is exactly as before, except that we replace $(6'), (7')$ with

$$\delta^h + \tilde{\delta}^h \leq \Delta(2') + \sum_{\ell \in L} p_{\ell m} q^h_{\ell m} + \sum_{\ell \in L_2} K p_{\ell c} q^h_{\ell c}$$

When netting is permitted, we need to strengthen the gains-to-trade hypothesis somewhat in order to guarantee the existence of credit card equilibrium..

Define the set H^* of households who have credit limits, i.e.,

$$H^* = \{h \in H : C^h < \infty\}.$$

Define $\overline{\mathcal{A}}$ to be the allocations in which no household trades cash-only commodities, and no credit-constrained household trades, i.e., $\overline{\mathcal{A}} = \{x \equiv (x^h)_{h \in H} \in \mathcal{A} : x^h_\ell = e^h_\ell$ for $h \in H$, $\ell \in L_1$; and $x^h = e^h$ for $h \in H^*\}$

Strengthened Intratemporal Gains to Trade Hypothesis For all $x \in \overline{\mathcal{A}}$, $\gamma(x) > \bar{m}/M$

Theorem 8 *Consider an economy* $(\mathcal{E}, C, \lambda, \tilde{\lambda}, m, M, L_1)$ *with* $\bar{m} > 0$. *Suppose the strengthened intratemporal gains-to-trade hypotheses holds. Then a credit card monetary equilibrium, with netting permitted, exists.*

Theorem 8 says that if there are cash-only commodities (not accessible to credit cards), or if there are households with credit limits, then money can still have value even when netting is permitted on credit cards. But as the number of such commodities and households is diminished, it becomes more difficult to sustain the value of money, since the set $\overline{\mathcal{A}}$ expands and the strengthened gains-to-trade hypothesis is rendered progressively more demanding.

For sufficiently high values of λ, $\tilde{\lambda}$ there is no default in an ME. This is because the presence of outside money puts a lower bound on cash prices $p_{\ell m}$ (otherwise any h with $m^h > 0$ could acquire arbitrary amounts of goods, a contradiction). Since $p_{\ell c} \geq p_{\ell m}$ as we already saw, all prices have a lower bound. Thus there is a cap on the "utiles" that a dollar (borrowed from the credit card company, or the bank) can buy; and, once the λ or $\tilde{\lambda}$ are higher than this cap, defaults will simply not be worthwhile for any household to undertake at any monetary equilibrium.

Finally we note that if L_1 and H^* are both empty then no ME exists, i.e., money loses all value. Of course, in this scenario, Walras equilibria are achieved as pure credit card equilibria, with netting.

7 Proofs

7.1 Proof of Theorem 4

For the proof, we first show

Lemma *Let* $M^* = M + \sqrt{\bar{m}M + M^2}$*. Then*

$$1 + \frac{\bar{m}}{M^*} = \sqrt{1 + \frac{\bar{m}}{M}}$$

and

$$M^* = \frac{M + \bar{m}}{\sqrt{1 + \frac{\bar{m}}{M}}} + M$$

Proof

$$\begin{aligned} M^* &= M + \sqrt{M^2 + \bar{m}M} \\ &= M + M\sqrt{1 + \frac{\bar{m}}{M}} = M + \frac{M\left(1 + \frac{\bar{m}}{M}\right)}{\sqrt{1 + \frac{\bar{m}}{M}}} \\ &= M + \frac{M + \bar{m}}{\sqrt{1 + \frac{\bar{m}}{M}}} \end{aligned}$$

showing the second equality. Moreover, from the second equality,

$$\frac{M^*}{M} = 1 + \sqrt{1 + \frac{\bar{m}}{M}}$$

so

$$\frac{M^*}{M}\left(\sqrt{1 + \frac{\bar{m}}{M}} - 1\right) = 1 + \frac{\bar{m}}{M} - 1 = \frac{\bar{m}}{M}$$

so

$$M^*\left(\sqrt{1 + \frac{\bar{m}}{M}} - 1\right) = \bar{m}$$

so

$$\sqrt{1 + \frac{\bar{m}}{M}} = 1 + \frac{\bar{m}}{M^*}$$

■

Proof of Theorem 4 We begin with a pure monetary equilibrium $(p^*, r^*, ({}^*q^h, {}^*x^h)_{h\in H})$ *of* $(\mathcal{E}, m, M^*)$. (By Theorem 1, such an equilibrium must exist). From this we construct the corresponding credit card equilibrium for $(\mathcal{E}^c, m, M)$. Define

$$\begin{aligned} r &= \frac{\bar{m}}{M} \\ p_{\ell m} &= p^*_{\ell m} \\ p_{\ell c} &= (1+r^*)p^*_{\ell m}. \end{aligned}$$

These are the market prices for our candidate credit card equilibrium. We shall start by showing that these prices give any household h exactly the same trading opportunities $B^h_c(p, r)$ in the credit card economy as he enjoyed via $B^h(p^*, r^*)$ in the pure monetary economy.

Observe that by Theorem 1, $r^* = \bar{m}/M^*$, since r^* is the equilibrium interest rate of the monetary economy $(\mathcal{E}, m, M^*)$. Hence, by the lemma, and the definition of r,

$$1 + r^* = \sqrt{1+r}.$$

In the purely monetary equilibrium, with prices (p^*, r^*), a household can only purchase commodities via cash, either out of his endowment or after borrowing the money. In the latter case, he must raise the money to repay the bank by a contemporaneous sale of $(1 + r^*)$ dollar's worth of other goods for each dollar borrowed.

In the candidate credit card equilibrium, any household can spend its endowment of money in precisely the same way, on the cash commodity markets, since $p_{\ell m} = p^*_{\ell m}$.

But buying on credit in the credit card economy is slightly more complicated. A household has the option, as in the pure monetary economy, of borrowing a dollar from the bank and then spending it on goods, while simultaneously selling $(1 + r)$ dollars of goods to defray the bank loan at the end. However, this option is very bad, since $r > r^*$. There are two other ways of buying on credit, which equally achieve the more efficient transactions tax of r^*. He can sell ℓ for cash and buy k with a credit card, or he can sell ℓ against a credit card and buy k with cash.

In the first way, let h sell $(1 + r^*)/p_{\ell m}$ units of good ℓ for cash. Let him simultaneously buy $(1 + r^*)/p_{kc}$ units of k with his credit card. Since $p_{kc} = (1 + r^*)p^*_{km}$, he is purchasing the equivalent of one dollar's worth of good k (at the prices p^*_{km}) while selling $1 + r^*$ dollar's worth of good ℓ (at the prices $p^*_{\ell m} = p_{\ell m}$).

Alternatively, let h sell $(1+r)/p_{\ell c}$ units of good ℓ on the credit card market, while simultaneously buying $1/p_{km}$ units of k with a dollar borrowed from the bank. Since the bank interest rate is r, the sale defrays the debt incurred by the purchase. Once again the household is buying a dollar's worth of good k (at prices $p^*_{km} = p_{km}$) while selling $1+r^*$ dollar's worth of good ℓ (at prices $p^*_{\ell m} = [1/(1+r^*)]p_{\ell c} = (1/\sqrt{1+r})p_{\ell c}$). Note that

$$\frac{1+r}{p_{\ell c}} = \frac{1+r}{(1+r^*)p^*_{\ell m}} = \frac{1+r^*}{p^*_{\ell m}}.$$

Hence a household will never buy a unit of k at the kc market and deliver on this promise via borrowed (and hoarded) bank money. For in this case he borrows p_{kc} dollars to obtain one unit of k at kc, whereas he would have done better to spend the money instead at the km market buying $p_{kc}/p_{km} = \sqrt{1+r}$ units of k. Moreover, each household will spend all its (owned or borrowed) money at the cheaper-priced $(\ell m)_{\ell \in L}$ markets rather than the $(\ell c)_{\ell \in L}$ markets.

Next note that any household who spends borrowed money at the $(\ell m)_{\ell \in L}$ markets will choose to repay the loan via sales at the higher-priced $(\ell c)_{\ell \in L}$ markets, rather than at the $(\ell m)_{\ell \in L}$ markets. On the other hand, sellers at cash-commodity markets will always use all the cash from their sales to make deliveries on credit card purchases. (Since the cash can be so used, and $p_{kc} = \sqrt{1+r}p_{km} < (1+r)p_{km}$ for all $k \in L$, it would be clearly inoptimal to borrow money at interest r to purchase at the $(\ell m)_{\ell \in L}$ markets against this cash).

This concludes our demonstration that the opportunity set in the credit card economy with prices (p, r) is the same for each household as in the pure monetary economy with prices (p^*, r^*). All that remains is to display budget feasible actions $(q^h, x^h)_{h \in H}$ with $x^h = {}^*x^h$, for each $h \in H$, such that all markets clear. We shall give an infinite recursion whose limit defines these actions.

The recursion begins by making households spend their money endowments in the L cash markets, in the same proportion as their expenditures in the pure monetary equilibrium. Dividing total expenditure on each commodity ℓ by the price $p_{\ell m} = p^*_{\ell m}$ gives a "demand" for each commodity ℓ. In general, in any round t we begin with an incremental demand for goods in the cash market. We construct a matching supply (out of goods households would have sold in the $*$-monetary equilibrium, but have not yet sold). These sales generate income against which the households can charge expenditures on their credit cards in round t. Again we construct a matching supply in the credit card markets out of goods households would have sold at the $*$-monetary equilibrium. These sales in the credit market in round t generate income which support the demand for expenditures of $1/(1+r)$ the value of sales revenue in the credit card market in the cash market at $t+1$. The recursion then continues.

The key idea is that each round replicates a piece of the trading in the $*$-monetary equilibrium. At the end of each round t, markets clear and every household has exactly financed its credit card purchases with contemporaneous cash sales, and is carrying credit card receivables from that round which will be used to cover exactly the principal and interest on the cash expenditures it will make in round $t+1$. The process must converge because the aggregate cash revenue generated in period $t+1$ is always $(1/1+r)$ (the aggregate cash revenue generated in period t).

To be precise, let

$$\beta^h_\ell = \frac{{}^*q^h_{m\ell}}{\sum_{k=1}^L {}^*q_{mk}}$$

denote the fraction of his expenditures agent h devoted to commodity ℓ in the $*$-monetary equilibrium.

Begin the recursion by making each agent h spend all his cash endowment m^h on the cash markets, in the same proportion as his expenditures in the $*$-monetary economy. Formally, define

$$q^h_{m\ell}(1) \equiv \beta^h_\ell m^h \text{ for all } \ell \in L, \text{ all } h \in H.$$

Note that $q^h_{m\ell}(1) \leq {}^*q^h_{m\ell}$. Let borrowing $q^h_{nm}(1) \equiv 0$. Define

$$Q_{\ell m}(1) \equiv \frac{1}{p_{\ell m}} \sum_{h=1}^{H} q^h_{m\ell}(1)$$

as the demand for goods created by the expenditures in round (1). If $Q_{\ell m}(1) = 0$, set $q^h_{\ell m}(1) \equiv 0 \; \forall h \in H$.

Otherwise, define

$$q^h_{\ell m} = \frac{Q_{\ell m}(1)^* q^h_{\ell m}}{\sum_{i=1}^{H} {}^*q^i_{\ell m}}.$$

Note that $q^h_{\ell m}(1) \leq {}^*q^h_{\ell m}$, and that supply equals demand for each good.

We now proceed to the credit card market. The sales of goods just constructed in the cash market give the sellers income against which to make credit card purchases. Define

$$q^h_{c\ell}(1) \equiv \beta^h_\ell \sum_{k=1}^{L} p_{km} q^h_{km}(1), \text{ for all } \ell, h$$

Let

$$Q_{\ell c}(1) \equiv \frac{1}{p_{\ell c}} \sum_{h=1}^{H} q^h_{c\ell}(1)$$

be the demand for goods in the credit card market, created by the expenditures in round (1). If $Q_{\ell c}(1) = 0$, set $q^h_{\ell c}(1) = 0$ for all $h \in H$.

Otherwise, define

$$q^h_{\ell c}(1) \equiv \frac{Q_{\ell c}(1)[{}^*q^h_{\ell m} - q^h_{\ell m}(1)]}{\sum_{i=1}^{H}[{}^*q^i_{\ell m} - q^i_{\ell m}(1)]}.$$

Note that supply equals demand in the credit market in round (1).

Assume inductively that $((q^h_{m\ell}(t), q^h_{nm}(t), q^h_{\ell m}(t), q^h_{c\ell}(t), q^h_{\ell c}(t))_{h\in H}, Q_{\ell m}(t), Q_{\ell c}(t))_{\ell \in L}$ have been generated for $t = 1, ..., T$. We now define the same vector for $t = T+1$.

Define

$$q^h_{m\ell}(T+1) \equiv \frac{\beta^h_\ell}{1+r} \sum_{k=1}^{L} p_{kc} q^h_{kc}(T) \text{ for all } \ell \text{ and } h$$

and

$$q^h_{nm}(T+1) \equiv (1+r) \sum_{\ell=1}^{L} q^h_{m\ell}(T+1).$$

Let

$$Q_{\ell m}(T+1) \equiv \frac{1}{p_{\ell m}} \sum_{h=1}^{H} q^h_{m\ell}(T)$$

be the total demand for goods in the cash market in round $T+1$. If $Q_{\ell m}(T+1) = 0$, set $q^h_{\ell m}(T+1) = 0$ for all $h \in H$.

Otherwise, define

$$q^h_{\ell m}(T+1) \equiv \frac{Q_{\ell m}(T+1)[{}^*q^h_{\ell m} - \sum_{t=1}^{T} q^h_{\ell m}(t) - \sum_{t=1}^{T} q^h_{\ell c}(t)]}{\sum_{i=1}^{H}[{}^*q^i_{\ell m} - \sum_{t=1}^{T} q^i_{\ell m}(t) - \sum_{t=1}^{T} q^i_{\ell c}(t)]}.$$

These supplies in the cash market at $T+1$ generate the income for purchases in the credit card market at $T+1$. Define

$$q^h_{c\ell}(T+1) = \beta^h_\ell \sum_{k=1}^{L} p_{km} q^h_{km}(T+1).$$

Let

$$Q_{\ell c}(T+1) \equiv \frac{1}{p_{\ell c}} \sum_{h=1}^{H} q^h_{c\ell}(T+1)$$

be the demand for goods in the credit card market in round $T+1$. If $Q_{\ell c}(T+1) = 0$, set $q^h_{\ell c}(T+1) = 0$ for all $h \in H$.

Otherwise, define

$$q^h_{\ell c}(T+1) \equiv \frac{Q_{\ell c}(T+1)[{}^*q^h_{\ell m} - \sum_{t=1}^{T+1} q^h_{\ell m}(t) - \sum_{t=1}^{T} q^h_{\ell c}(t)]}{\sum_{i=1}^{H}[{}^*q^i_{\ell m} - \sum_{t=1}^{T+1} q^i_{\ell m}(t) - \sum_{t=1}^{T} q^h_{\ell c}(t)]}.$$

This concludes the recursion.

Define

$$q^h \equiv \sum_{t=1}^{\infty} q^h(t).$$

Let

$$x^h_\ell \equiv e^h_\ell + \frac{q^h_{m\ell}}{p_{\ell m}} + \frac{q^h_{c\ell}}{p_{\ell c}} - q^h_{\ell m} - q^h_{\ell c} \text{ for all } h, \ell.$$

Recall that we assume $\bar{m} > 0$, hence $r = \bar{m}/M > 0$. By monotonicity, we know $p^*_{\ell m} > 0$ for all $\ell \in L$. By construction, the value of aggregate purchases in the cash market at round 1 is $\bar{m} > 0$. This generates aggregate (matching) sales of value $\bar{m}$. These sales call forth credit card purchases of aggregate value $\bar{m}$ in round 1, again generating matching sales of aggregate value $\bar{m}$. In round 2, aggregate bank promises must be $\bar{m}$, so aggregate expenditures are only $\bar{m}/(1+r)$. Thus aggregate expenditures for cash and credit in any period t must be $2\bar{m}/(1+r)^{t-1}$, split equally between cash and credit.

This sum converges, and dominates the sales and purchases of each individual. Hence q^h is well-defined for each h.

Note that

$$2\sum_{t=1}^{\infty}\frac{\bar{m}}{(1+r)^{t-1}} = 2\frac{(1+r)\bar{m}}{r} = 2(M+\bar{m}),$$

since $rM = \bar{m}$. Hence $M+\bar{m}$ is spent for cash purchases, and $M+\bar{m}$ is spent for credit card purchases. But credit card prices $p_{\ell c} = (1+r^*)p_{\ell m} = (1+r^*)p^*_{\ell m}$. Consider the revenue generated by each sale (whether in the cash market or the credit card market) evaluated at the prices $p^*_{\ell m}$. Aggregated over all households and commodities, this is

$$M + \bar{m} + \frac{M+\bar{m}}{1+r^*}.$$

From our lemma, this is equal to

$$M^* + \bar{m}.$$

Since supply equals demand by construction, total expenditures (evaluated at prices p^*) must also be $M^* + \bar{m}$. But also by construction, each agent is always making trades which would balance his budget at p^* in the monetary economy, without every exceeding total sales ${}^*q^h_{\ell m}$. Hence we must in fact have that

$$q^h_{\ell m} + q^h_{\ell c} = {}^*q^h_{\ell m}$$

and

$$x^h = {}^*x^h.$$

For the converse part of the theorem, start with a credit card equilibrium $(p, r, (q^h, x^h)_{h\in H})$ *of* $(\mathcal{E}^c, m, M)$. Define

$$\begin{aligned}
r^* &= \frac{\bar{m}}{M^*} \\
p^*_{\ell m} &= p_{\ell m} \\
{}^*q^h_{\ell m} &= q^h_{\ell m} + q^h_{\ell c} \\
{}^*q^h_{m\ell} &= p_{\ell m}(x^h_\ell - e^h_\ell)^+ \\
{}^*q^h_{nm} &= (1+r^*)(\sum_{\ell\in L}^{*} q^h_{m\ell} - m^h) \\
{}^*x^h_\ell &= x^h_\ell
\end{aligned}$$

It is straightforward to verify that we have defined a pure monetary equilibrium.

■

7.2 Proof of Theorems 6 and 8

We shall prove Theorems 6 and 8 simultaneously, indicating what modifications need to be made if netting is allowed as in Theorem 8.

If $C^h = 0 \ \forall h \in H$, then Theorems 6 and 8 are identical to the no credit card theorem, i.e., to Theorem 1. Hence we suppose some $C^h > 0$.

For any $\epsilon > 0$, we shall define a generalized game Γ^ϵ with a finite-type continuum player set. Replace each $h \in H$ by a continuum of identical players $t \in (h-1, h]$ (i.e., $e^t \equiv e^h, u^t \equiv u^h, C^t \equiv C^h, \lambda^t \equiv \lambda^h, \tilde{\lambda}^t \equiv \tilde{\lambda}^h$ for $t \in (h-1, h]$). Given a measurable choice of market actions $q_{\alpha\beta} : (0, H] \to \mathbb{R}_+$ and $q_{\beta\alpha} : (0, H] \to \mathbb{R}_+$ at the market $\alpha\beta$, the price $p_{\alpha\beta}(\epsilon)$ is formed in Γ^ϵ simply as the ratio of aggregate β to aggregate α. But we suppose there is an external dummy who puts up ϵ units on each side of each market, so that

$$p_{\alpha\beta}(\epsilon) = \frac{\epsilon + \int_0^H q^t_{\beta\alpha} dt}{\epsilon + \int_0^H q^t_{\alpha\beta} dt}$$

except on the credit card/commodity markets,where it puts up only ϵ^2 by way of commodity sales, so that

$$p_{\ell c}(\epsilon) = \frac{\epsilon + \int_0^H q^t_{c\ell} dt}{\epsilon^2 + \int_0^H q^t_{\ell c} dt}$$

The ϵ^2 ensures that if the external dummy is the sole seller of commodities on the ℓc market, then $p_{\ell c}(\epsilon) \to \infty$ as $\epsilon \to 0$. Furthermore, we suppose that the external dummy fully delivers on his credit card promises (thus creating $|L_2|\epsilon$ units of money for delivery). Thus the delivery rate in Γ^ϵ is

$$K(\epsilon) = \frac{|L_2|\epsilon + \int_0^H \delta^t dt}{|L_2|\epsilon + \sum_{\ell \in L_2} \int_0^H q^t_{c\ell} dt}$$

The set of feasible actions of any player depends on others' actions only via the $p_{\alpha\beta}(\epsilon)$ and $K(\epsilon)$, and is defined exactly as in $(1')$ till $(8')$, except that we impose an upper bound of $1/\epsilon$ on all actions of the players in Γ^{ϵ}. Payoffs are defined exactly as before (with or without netting). This completely specifies the generalized game Γ^{ϵ}.

By an ε-ME, we shall mean a type-symmetric Nash equilibrium (in pure strategies) of Γ^{ϵ}. It exists by the standard Nash argument. (On account of type-symmetry, we may write $\sum_{h=1}^{H}, q_{\alpha\beta}^{h}, \delta^{h}$ instead of $\int_{0}^{H}, q_{\alpha\beta}^{t}dt, \delta^{t}dt$, respectively, at an ϵ-ME.)[17]

Consider a subsequence of ε-ME as $\varepsilon \to 0$ in which all quantities, as well as their ratios, are converging (possibly to infinity or zero).

For small ε, the external dummy is creating less than 1 unit of goods or money. Letting $\tilde{M} = M+\bar{m}+1$, any household h will end up with utility at most $u^{h}((1,...,1)+\sum_{i\in H} e^{i}) - \tilde{\lambda}^{h}(q_{nm}^{h}(\varepsilon) - \tilde{M}) - \lambda^{h}(\sum_{\ell\in L_2} q_{c\ell}^{h}(\varepsilon) - \tilde{M})$. We shall often use the fact that since utilities are concave and strictly increasing, we may assume without loss of generality that $u^{h}(x) \to \infty$ as $x_k \to \infty$ for any component k, as explained in Dubey-Geanakoplos (2003).

We can see from the above upper bound on utility that the bonds promised to the bank ($q_{nm}^{h}(\varepsilon)$) will be bounded in spite of the possibility of default, since otherwise h would have done better not trading and obtaining utility $u^{h}(e^{h})$. Similarly $q_{c\ell}^{h}(\varepsilon)$ will be bounded when there is no netting. It is obvious that cash expenditures $q_{m\ell}^{h}$ cannot exceed the total cash on hand, which is bounded by $\tilde{M}$, and commodity sales $q_{\ell m}^{h}, q_{\ell c}^{h}$ cannot exceed the endowments e_{ℓ}^{h}. Hence all quantities have finite limits.

Clearly the gross interest rate $(1 + r(\varepsilon))$ cannot fall below 1 in any $\varepsilon - ME$, otherwise agents in the tremble would borrow infinitely, spend the fraction $-r(\varepsilon)$ on goods, obtaining arbitrarily large utility, and still repay the bank, contradicing the upper bound on utility given above. The interest rate must also stay bounded above, on account of the boundedness of $q_{nm}^{h}(\varepsilon)$.

All prices $p_{\ell m}(\varepsilon)$ stay bounded away from zero, for otherwise any agent with $m^{h} > 0$ could buy arbitrarily large amounts of good ℓ contradicting the upper bound on utility. So do credit card prices. If $p_{\ell c}(\varepsilon) \to 0$ then any agent with $C^{h} > 0$ can buy $\min\{C^{h}, \sum_{k\in L} p_{km}(\varepsilon)e_{k}^{h}\}/p_{\ell c}(\varepsilon) \to \infty$ of good ℓ with his credit card, and repay the credit card from the receipts from the sales of his endowment on the cash markets, again contradicting the upper bound on utility.

Suppose some price ratio $p_{\ell m}(\varepsilon)/p_{kx}(\varepsilon) \to \infty$ (where $x = m$ or c) as $\varepsilon \to 0$. Then any household h with $e_{\ell}^{h} > 0$ can sell e_{ℓ}^{h} for cash $p_{\ell m}(\varepsilon)e_{\ell}^{h}$, borrow $p_{\ell m}(\varepsilon)e_{\ell}^{h}/(1+r(\varepsilon))$ dollars, and buy $p_{\ell m}(\varepsilon)e_{\ell}^{h}/[(1+r(\varepsilon))p_{kx}(\varepsilon)] \to \infty$ of commodity k. If $x = m$, the agent buys with the borrowed money, and defrays the bank loan from the sales of e_{ℓ}^{h}. If $x = c$, the agent repays his credit card purchase with the borrowed bank money and later defrays the bank loan with the proceeds of the sale of e_{ℓ}^{h}. Then, for sufficiently small ϵ, the agent will achieve utility exceeding the upper bound, a contradiction. Hence if any cash price goes to infinity, so do all prices.

[17] The symbols $q_{\alpha\beta}^{h}, \delta^{h}$ will refer equally to aggregate or individual actions. The meaning will always be clear from the context.

Next observe that if a credit card price $p_{\ell c}(\varepsilon) \to \infty$ then for small ε no buyer on that market will default, since he gets virtually no goods for his dollar but pays a significant penalty λ. Since the external dummy always keeps his promises, $K(\varepsilon) = 1$. But now precisely the same argument given in the previous paragraph shows that all prices must go to infinity.

To sum up, if any price goes to infinity, so do all prices.

We shall now show that prices do not go to infinity. Suppose they do. Then all sales of commodities on cash markets or credit card markets must go to zero, since we have shown that expenditures are bounded above. That implies final consumption converges to the initial endowments. Without netting, this contradicts the gains to trade hypothesis (as explained in Dubey-Geanakoplos 2003). If there is netting, then we contradict the strengthened gains to trade hypothesis.

It is easy to check that the requirements of refinement are met. Observe that since prices do not go to infinity, for each ℓ there exists some seller h with $q^h_{\ell c}(\varepsilon) > 0$, for otherwise $p_{\ell c}(\varepsilon) \geq \varepsilon/\varepsilon^2 \to \infty$. (By the way, if $K(\varepsilon) < 1$, then since the external agent always redeems his debts, there must be some buyer who is defaulting, that is some h and commodity ℓ with $q^h_{c\ell}(\varepsilon) > 0$.)■

7.3 Proof of Theorem 7

Define a reallocation cycle c_n as a sequence of distinct commodities $(\ell_1, ..., \ell_n)$ and households $(h_1, ..., h_n)$. By Theorem 1 in Dubey-Geanakoplos (2003),

$$1 + \gamma(x) = \max_{2 \leq n \leq L} \max_{c_n \in C_n} \{ \times_{i=1}^{n} \frac{\frac{\partial u^{h_i}}{\partial x_{\ell_{i+1}}}(x^{h_i})}{\frac{\partial u^{h_i}}{\partial x_{\ell_i}}(x^{h_i})} \}^{\frac{1}{n}}$$

where the second max is taken over the finite set C_n of all reallocation cycles of length n.

In view of this formula, the first equality will follow if we can show that the product is at most $\sqrt{(1 + \bar{m}/M)/K\tilde{K}}$ for every cycle, and exactly that high for at least one cycle.

Since the CCE is interior, there exists a cycle c^* in which h_i is selling ℓ_i against both cash and credit card, and is also buying ℓ_{i+1} against both cash and credit card. To buy ϵ units more (or less) of ℓ_{i+1} with cash obtained from a bank loan, h_i must sell $\epsilon(1 + r)p_{\ell_{i+1}m}/Kp_{\ell_i c}$ units more (or less) of ℓ_i on the credit card market in order that the debt to the bank remains invariant. Therefore

$$\frac{\partial u^{h_i}}{\partial x_{\ell_{i+1}}}(x^{h_i}) = \frac{(1 + r)p_{\ell_{i+1}m}}{Kp_{\ell_i c}} \frac{\partial u^{h_i}}{\partial x_{\ell_i}}(x^{h_i})$$

(If h were not buying ℓ_{i+1} or not selling ℓ_i then we would have the $LHS \leq RHS$).

Similarly, to buy ϵ units more (or less) of ℓ_{i+1} with his credit card, h_i must sell $\epsilon p_{\ell_{i+1}c}/p_{\ell_i m}$ more (or less) of ℓ_i on the cash market in order that his credit card debt remains invariant. Therefore

$$\frac{\partial u^{h_i}}{\partial x_{\ell_{i+1}}}(x^{h_i}) = \frac{p_{\ell_{i+1}c}}{p_{\ell_i m}} \frac{\partial u^{h_i}}{\partial x_{\ell_i}}(x^{h_i})$$

(If h were not buying ℓ_{i+1} or not selling ℓ_i then we would have the $LHS \leq RHS$).

Multiplying the corresponding sides of these two displays and rearranging terms and taking the square root of both sides, we have

$$\frac{\frac{\partial u^{h_i}}{\partial x_{\ell_{i+1}}}(x^{h_i})}{\frac{\partial u^{h_i}}{\partial x_{\ell_i}}(x^{h_i})} = \sqrt{\frac{(1+r)p_{\ell_{i+1}m}p_{\ell_{i+1}c}}{Kp_{\ell_i m}p_{\ell_i c}}}$$

Since no one will hold worthless cash at the end, all the money in the system will be delivered to the bank, i.e. $\sum \tilde{\delta}^h = M + \bar{m}$. So $\tilde{K}(\sum_{h \in H} q^h_{nm}) = M + \bar{m}$. From the fact that $(1+r)M = \sum_{h \in H} q^h_{nm}$, we conclude that $(1+r) = (1 + \frac{\bar{m}}{M})/\tilde{K}$. Thus the RHS of the last display is

$$\sqrt{\frac{(1+\frac{\bar{m}}{M})p_{\ell_{i+1}m}p_{\ell_{i+1}c}}{K\tilde{K}p_{\ell_i m}p_{\ell_i c}}}$$

for every h_i in the cycle c^*. The n^{th} root of the product of these RHS over all n households in the cycle is just

$$\sqrt{\frac{(1+\frac{\bar{m}}{M})}{K\tilde{K}}}$$

So we have produced one cycle that meets the bound. Every other cycle must do the same or worse, since, as we have noted, there would then be inequalities and not equalities.

This proves the first equality. The second equality follows from the fact (already noted) that $\frac{(1+\frac{\bar{m}}{M})}{\tilde{K}} = 1 + r$. It remains to argue the last inequality. Clearly

$$Kp_{\ell_i c} \geq p_{\ell_i m}$$

otherwise h_i would not sell ℓ_i on the ℓ_{ic} market (it is strictly better for him to transfer the sale to the ℓ_{im} market). Hence, from the second display, we get

$$\frac{\partial u^{h_i}}{\partial x_{\ell_{i+1}}}(x^{h_i}) \leq \frac{(1+r)p_{\ell_{i+1}m}}{p_{\ell_i m}} \frac{\partial u^{h_i}}{\partial x_{\ell_i}}(x^{h_i})$$

for all h_i in the cycle c^*. Taking products and then the nth root, we have

$$\{\times_{i=1}^{n} \frac{\frac{\partial u^{h_i}}{\partial x_{\ell_{i+1}}}(x^{h_i})}{\frac{\partial u^{h_i}}{\partial x_{\ell_i}}(x^{h_i})}\}^{\frac{1}{n}} \leq 1 + r.$$

But the LHS was already shown equal to $\sqrt{(1+r)/K}$.

■

8 References

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