A spatial hybrid motion compliant mechanism: Design and optimization

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A hybrid motion system is defined as a mechanical system that combines a macro motion and a micro motion into one system to achieve a large motion and high resolution with fast response simultaneously. In this paper, a spatial hybrid motion mechanism with 3-DOFs is developed that integrates two types of motion through only one compliant mechanism: a macro motion driven by DC servomotors for large workspace and a micro motion driven by PZT actuators for high precision. A unique feature of the developed hybrid motion compliant mechanism is the elimination of coupling interaction between the macro motion and the micro motion by properly structure design. Three issues are addressed in this paper: (1) design principle and implementation of the hybrid motion mechanism; (2) kinematic analysis and dynamic analysis; and (3) optimization design of the hybrid motion mechanism. A spatial hybrid motion mechanism is developed and the optimization is conducted. The Taguchi method is used to identify significant parameters in the design optimization, and finite element analysis results verify the design principle of the parallel architecture for the hybrid motion mechanism.

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1. Introduction

High precision manipulation systems with large range of motion and high positioning resolution are highly needed for industrial applications such as various machining processes, cell manipulation, and computer component assembly. A possible system that can satisfy these requirements is a macro/micro manipulator system [1–5]. In such a system, a long reach, or macro, manipulator is characterized by a large workspace with slow response due to its size. In contrast, a short reach, or micro, manipulator is characterized by a small work volume with fast and precise manipulation capability over that work volume. Such a macro/micro mechanism can be called a hybrid motion mechanism (HMM). The concept of a macro/micro manipulator was first introduced by Sharon et al. [1]. A macro/micro manipulator test-bed was developed in [2] where a 2-DOFs macro manipulator with two flexible links is attached by a 3-DOFs small robot. A macro/micro system integration using a parallel kinematic mechanism and a 2-DOFs micro manipulator for the application of deburring and finishing operations was developed in [3]. But the interaction issue between micro and macro mechanisms was not taken into consideration. An 11-axis robot to accomplish accurate positioning and velocity-controlled tasks in the presence of a flexible substructure was designed [4] through a macro/micro system combination.

As mentioned in [5], mounting a micro manipulator on a macro manipulator would produce a dynamic interaction problem that might degrade performances of the whole system. Therefore, many efforts were focused on controller design for macro/micro motion systems [6–8], rather than on the mechanism design. A hybrid position/force control of flexible manipulators by a macro/micro manipulator system was proposed in [6]. In that design, a macro manipulator was controlled to roughly realize the desired position and force by a simple PD feedback, and a micro manipulator was used to compensate the position and force errors caused by the macro part. A contact control for a flexible macro/micro manipulator was discussed in [7] where the controller combined force damping control and inertial force active damping control. Dynamic control of a flexible macro/micro manipulator system was studied in [8] where a PD feedback controller was employed incorporating a fuzzy adaptive tuner for the macro manipulator, and a control scheme was proposed for the micro manipulator.

It should be noted that all the architectures of the developed macro/micro manipulator systems are connected in series. It means that a micro manipulator is mounted on the tip of a macro manipulator. A mechanism based on such architecture can be called a serial HMM. The majority of the HMM discussed in the literature was focused on the attenuation of vibration of the macro manipulator caused by a fast action of the micro manipulator. From the above discussion, we can see that traditional macro/micro motion systems are combined simply using two separately designed motion stages. Such a design is easy to realize, but the assembly error and the backlash force will affect the accuracy of the end-effector in the combined system. Furthermore, mounting...
a micro manipulator at the tip of a macro manipulator results in a dynamically coupled system, which increases the complexity of control system design.

The micro motion of a traditional HMM will affect the macro motion, and further affects the system performance. The interaction between the macro motion and the micro motion is a big issue in the design and control of HMMs. It is a challenge to solve the interaction problem for a traditional HMM in a two-stage design and little progress was achieved. That is the main motivation for us to develop a novel spatial HMM in this paper. There are some researches focusing on dual-stage actuation system design and control [9–13]. It should be noticed that most of the developed dual-stage actuation systems are 1-DOF linear motion systems through simple connection of a macro actuator and a micro actuator. These systems did not solve the interaction problem in a design point of view. There is no multi-DOFs HMM driven by two different actuators in the literature. In this paper, a new design strategy for a spatial HMM based on a compliant mechanism is proposed. The main idea behind the new design is that two types of actuators, i.e., a DC servomotor for the coarse motion and a PZT actuator for the fine motion, are integrated to form one actuating motion that is called hybrid motion. The novelty of the new design is the elimination of the interaction between a coarse motion and a fine motion through mechanical design and actuator arrangement. In our design, only one mechanism is enough to complete coarse and fine positioning performance, which is the big difference between the proposed design method and a traditional one.

The Taguchi method is a fractional factorial design of experiments [14] where the orthogonal array (OA) is used as a tool to minimise the number of function evaluations. A performance obtained from the experiments is taken as quality characteristics in searching for optimal settings of design parameters at which the performance is the best [15–18]. The Taguchi method can be used in the optimization of structural design. In our study, the Taguchi method is used to optimize the structural parameters associated with the flexural hinges of the designed HMM for the purpose of maximizing the performance of the HMM.

The reminder of this paper is organized as follows. In Section 2, a spatial compliant mechanism with 3-DOFs is designed to form a HMM that is based on a compliant mechanical amplifier. Specifically, the kinematics analysis is conducted and the inverse kinematics is highlighted. Section 3 conducts static and dynamic analyses for different configurations of the HMMs using finite element analysis (FEA) software. Section 4 discusses structure optimization of the HMM using the Taguchi method based on several objective functions. Finally, some conclusions and future work are summarized.

2. Design and modeling of a spatial HMM

In many manipulation applications, such as cell DNA injection manipulation, it is required to orientate the injecting needle in a large range (about $10^6$) and high positioning accuracy (at the sub micro degree) to facilitate a successful injection manipulation [18]. To reduce the effect of an external disturbance under operating conditions, the natural frequency should be relative high (>400 Hz in this research). A compact structure is needed for the designed HMM in order to be integrated to an automated cell manipulation system. The goal of this research is to develop a HMM that can fulfill such requirements. It is very difficult for a traditional joint connected spatial mechanism to accomplish such high performances. In this section, a new design approach for a spatial HMM is proposed based on a compliant mechanism [19] to satisfy these requirements. More specifically, the HMM is constructed based on a developed compliant mechanical amplifier (CMA) [20] that can enlarge the PZT stroke and maintain high accuracy.

2.1. Design principle for HMM

Traditionally, the design of a HMM is divided into two separate motion stages. One is for the coarse motion that functions as a base with a large range of motion, while the other is for the fine motion with high accuracy which is mounted on the coarse motion stage. Such a HMM can be viewed as a serial configuration and its topology can be expressed in Fig. 1a. From Fig. 1a, we can see that the macro motion driven by a macro driver will affect the micro motion actuated by a micro driver, and the micro motion will affect the macro motion as well. In the control point of view, the effect between the macro motion and the micro motion is called dynamic coupling or interaction.

To overcome the dynamic interaction problem associated with the traditional HMM, a new design strategy is proposed that can be viewed as a parallel configuration and the topology is shown in Fig. 1b. Such a parallel design (see Fig. 2a) can also be called a hybrid actuation design. In the parallel configuration, the macro motion and micro motion are integrated and drive the same compliant mechanism. Only one mechanism is needed to produce the coarse (macro) and fine (micro) motion. Such a characteristic can be realized using a compliant mechanism with flexural hinges. The compliant mechanism can obtain smooth and continuous motions, high resolution, and reduced backlash [19]. All these features are necessary for developing a HMM used in cell manipulation. In the next section, details of design and analysis of the HMM are discussed.

2.2. Design and analysis of HMM

2.2.1. Design of HMM

The proposed spatial HMM is a parallel type compliant mechanism with three identically serial chains or legs shown in Fig. 2. Each leg consists of a 1-axis flexural hinge above point B as revolute joint and a 3-axis flexural hinge below the upper plate as a spherical joint.

In the designed HMM, a CMA designed for the amplification of a PZT actuator [20] is applied to amplify the micro motion and change the motion direction to eliminate the interaction between the micro motion and the macro motion. The driving force produced by a PZT is on the horizontal direction and transferred through the CMA to the vertical direction, and the macro motion driven by DC motors from the bottom of each leg is on the vertical direction; see Fig. 2. Such an arrangement can implement the parallel HMM, which can be explained as follows. First, from the viewpoint of the end-effector point P, the mechanical structure is apparently parallel, especially with respect to the three identical legs. All three hybrid motions generated by three legs are connected directly to the same end-effector P. The motion generated and transferred to the end-effector is a mixed macro and micro motion through the flexural hinges. Second, from the viewpoint of point B which is an output portal of the CMA mechanism, its motion is a mixed macro and micro motion synchronously contributed by the PZT actuator and the DC motor. Accordingly to the pseudo-rigid-body model (PRBM) of a compliant mechanism [19], the CMA can be schematically described, as shown in Fig. 2b. The output of CMA in Fig. 2b can be controlled by a PZT for a micro high precision motion and a DC motor for a macro motion.

According to the designed structure shown in Fig. 2a, the developed HMM has the following characteristics.

(1) The mechanism is a symmetric compliant mechanism with 3-DOFs due to three identical legs.
(2) The moving plate and the base plate are designed to be equilateral triangles.
(3) A CMA is applied to increase the stroke of the PZT actuator for the micro motion.
(4) DC servomotors are used to directly drive the base of the CMAs to provide the macro motion.
(5) The interaction between DC servomotors and PZT actuators is eliminated by the orthogonal arrangement of the drivers and the symmetric structure of the CMA.
(6) The designed HMM can be used as a micro motion mechanism if only PZT actuators are used to drive three legs.

2.2.2. Transformation matrix

In order to conduct kinematic analysis of the HMM, a transformation matrix based on the defined base plate and moving plate coordinate systems in Fig. 3 is needed. To get the transformation matrix of the local coordinate attached to the end plate with respect to the global coordinate, the $X$-$Y$-$Z$ Euler angles $(\alpha, \beta, \gamma)$ reference system [21] is used.

The matrix representing the Euler angles orientation can be expressed as:

$$\text{Euler}(\alpha, \beta, \gamma) = R_z(\gamma)R_y(\beta)R_z(\alpha)$$

where

$$\begin{bmatrix}
    n_x \\
    n_y \\
    n_z
\end{bmatrix} = 
\begin{bmatrix}
    \cos \beta \cos \gamma \\
    \sin \alpha \sin \beta \cos \gamma + \cos \alpha \sin \gamma \\
    - \cos \alpha \sin \beta \cos \gamma + \sin \alpha \sin \gamma
\end{bmatrix}$$

$$\begin{bmatrix}
    o_x \\
    o_y \\
    o_z
\end{bmatrix} = 
\begin{bmatrix}
    - \cos \beta \sin \gamma \\
    - \sin \alpha \sin \beta \sin \gamma + \cos \alpha \cos \gamma \\
    - \cos \alpha \sin \beta \sin \gamma + \sin \alpha \cos \gamma
\end{bmatrix}$$

Fig. 1. Two design principles for HMM.

Fig. 2. 3D model of the spatial HMM.
where \( w \) with parameters (the HMM, the position vector of the three independent variables. Due to the symmetric design of the three dependent variables should be represented by functions.

2.2.3. Solution of the dependent parameters

To obtain an explicit expression of the transformation matrix, the three dependent variables should be represented by functions of the three independent variables. Due to the symmetric design of the HMM, the position vector \( c_i \) of the point \( C_i (i = 1, 2, 3) \) in Fig. 3a with respect to the local coordinate system \( XbYbZb \) can be described as

\[
\begin{bmatrix}
    c_1 \\
    0 \\
    -c_2
\end{bmatrix}
\]

\[
\begin{bmatrix}
    -\frac{1}{2}c_x \\
    \frac{\sqrt{3}}{2}c_x \\
    -c_2
\end{bmatrix}
\]

\[
\begin{bmatrix}
    -\frac{1}{2}c_x \\
    -\frac{\sqrt{3}}{2}c_x \\
    -c_2
\end{bmatrix}
\]

(6)

with

\[
\begin{bmatrix}
    c_x = r_0 + L_4 \cos \psi \\
    c_2 = L_4 \sin \psi
\end{bmatrix}
\]

(7)

where \( \psi \) is defined in Fig. 3b.

Using the transformation Eq. (5), the position vector \( c_i \) can also be described with respect to the global coordinate system \( XpYpZp \) as follows.

\[
\begin{bmatrix}
    C_i \\
    1
\end{bmatrix}
= T_p^b \begin{bmatrix}
    c_i \\
    1
\end{bmatrix}
\]

for \( i = 1, 2, 3 \)

(8)

As the links \( B_1C_1, B_2C_2, \) and \( B_3C_3 \) are constrained into one, we have

\[
\begin{bmatrix}
    C_{1y} = 0 \\
    C_{2y} = -\sqrt{3}c_x \\
    C_{3y} = -\sqrt{3}c_x
\end{bmatrix}
\]

(9)

Applying Eq. (9) to Eq. (8), we can get

\[
\begin{bmatrix}
    n_y = o_x \\
    x_p = -\frac{1}{2}(o_x c_x - n_x c_x - 2a_x c_z) \\
    y_p = a_x c_x - n_y c_x
\end{bmatrix}
\]

(10)

(11)

(12)

Submitting Eqs. (2)–(4) into Eq. (11), the Euler angle \( \gamma \) is obtained as:

\[
\gamma = \arctan\left(-\frac{\sin \alpha \sin \beta}{\cos \alpha + \cos \beta}\right)
\]

(13)

2.2.4. Inverse kinematics

The problem of inverse kinematics is to determine the motion values of the actuated drivers from the desired position and orientation (\( x, y, z, \alpha, \beta, \) and \( \gamma \). As this HMM has 3-DOFs, we assume the three independent parameters are \( \alpha, \beta, \) and \( z_p \), then the other three parameters (\( x_p, y_p, \) and \( \gamma \)) are the dependent parameters.

1. Obtaining the Euler angle \( \gamma \) from Eq. (13).
2. Calculating transformation matrix in Eq. (1) using Eqs. (2)–(4).
3. Obtaining \( c_x \) and \( c_z \) from Eq. (7).
4. Getting position vector \( (x_p, y_p) \) using Eqs. (11) and (12).
5. Obtaining the \( C_{ix}, C_{iy}, C_{iz} (i = 1, 2, 3) \) using Eq. (8).
6. Calculating the \( B_{xi}, B_{yi}, B_{zi} \) (i = 1, 2, 3) according to the global coordinate system in Figs. 3 and 4.
7. Using the following equation to obtain the required displacement \( d_i (i = 1, 2, 3) \) for the macro/micro motion.

\[
d_i = C_{ix} - L_0 - L_1 - \left(L_{BC}^2 - (C_{ix} - B_{xi})^2 - (C_{iy} - B_{yi})^2\right)^{0.5}
\]

(14)

where \( L_{BC} \) is the Euclidean distance between joint \( B \) and joint \( C \).
3. Motion prediction and dynamic analysis

3.1. Design parameters

According to the schematic of the HMM shown in Fig. 2, two types of spatial HMMs are designed that use different flexural hinges as spherical joints with three rotational motions. One is called a square type HMM where the 3-axis flexural hinges with a square cross-section are used; the other is called a cylinder type HMM where the 3-axis flexural hinges with a cylindrical cross-section are used. These two types of flexural hinges can be viewed as ball or spherical joints with the ability of three rotations, enabling very smooth movements with extremely high accuracy.

To compare performances of these two types of HMMs, two different configurations with different design parameters are used first to predict the orientation angle of the moving plate of the HMMs. Some design parameters that significantly affect the performances of the HMM are identified according to the compliant mechanism theory and the conducted kinematic analysis in Section 2. Some fixed parameters and some design parameters for a leg of the HMM are shown in Fig. 4. The overall dimension of the HMM is selected according the requirements discussed in Section 2. In the design process, eleven design parameters associated to the flexural joints are chosen and listed in Table 1.

In this paper, a 3D finite element model using ANSYS software is built to calculate the static and dynamic performances of the HMMs. A mixed meshing process (auto meshing and manual meshing methods shown in Fig. 5) is used in the finite element model so that the number of total nodes will not be too large and the processing time can be reduced. The key in this meshing process is that meshing for the rigid bodies is rough and meshing for the flexural hinges is fine so that a more accurate analysis result can be obtained. In the following parts, the static and dynamic analyses of two designed HMMs are conducted to get some primary knowledge as the base for the optimal design of the HMM being discussed in Section 4.

3.2. Orientation prediction

Based on the symmetric arrangement of the actuations in the HMM, one can see that the translation motion in the Z direction can be easily achieved by simultaneously moving the three legs using three DC motors to achieve the macro motion, or by actuating three PZTs to provide the micro motion. Therefore, only the orientation of the HMM is presented in this paper.

In FEA, the 3D element Solid185 is selected to model the 3D structure of HMMs. The modulus of the elasticity $E$ of the material is set to $7.5 \times 10^{10}$ N/m$^2$ and the Poisson ratio $\mu = 0.33$. In the following, the HMM with a square sectional spatial hinge is called a square type HMM, while the HMM with a cylindrical cross-section spatial hinge is called a cylinder type HMM. These two types of HMMs with two different configurations are analyzed under micro motion, macro motion, and macro/micro motion conditions. Fig. 5 shows the micro motion scheme of the HMM driven by two PZTs and Fig. 6 shows the stress profile of the HMM for a micro motion.

From Fig. 6 one can see that the maximum stress of the HMM

Table 1

<table>
<thead>
<tr>
<th>Parameters</th>
<th>$h$</th>
<th>$l$</th>
<th>$t$</th>
<th>$d$</th>
<th>$R_0$</th>
<th>$R_1$</th>
<th>$L_2$</th>
<th>$L_3$</th>
<th>$L_4$</th>
<th>$L_5$</th>
<th>$F_s$</th>
<th>$R_s$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case 1</td>
<td>1.0</td>
<td>1.5</td>
<td>0.3</td>
<td>0.4</td>
<td>0.3</td>
<td>0.5</td>
<td>6.0</td>
<td>10.0</td>
<td>1.0</td>
<td>0.5</td>
<td>1.0</td>
<td></td>
</tr>
<tr>
<td>Case 2</td>
<td>1.0</td>
<td>2.0</td>
<td>0.4</td>
<td>0.5</td>
<td>0.4</td>
<td>0.6</td>
<td>10.0</td>
<td>14.0</td>
<td>3.0</td>
<td>0.5</td>
<td>1.0</td>
<td></td>
</tr>
</tbody>
</table>
occurs at the 1-axis flexural hinges of the CMA. It should be noted that the maximum stress of the HMM for the macro motion driven by DC motors occurs at the 3-axis spatial flexural hinge of the driving leg.

In the design process, the predicted orientation (rotation around the Y axis), the driving force and the maximum stress of the HMMs are obtained for two cases under the micro motion, macro motion and macro/micro motion situations, respectively. Table 2 lists the micro orientation results driven by one PZT and two PZTs, respectively; Table 3 shows the macro orientation results, and Table 4 lists the hybrid orientation results, respectively. From these three tables, the following conclusions can be drawn. First, the orientation angle of the HMMs driven by one leg is the same but in the opposite direction as driven by two other legs. This is due to the symmetric design of the three identical legs. Second, the cylinder type HMM has the potential of a larger orientation motion than the square type HMM. This implies that the spatial flexural hinge with a cylindrical cross-section has a larger compliance than the one with a square cross-section.

Focusing on the macro/micro orientation, the required driven force for PZTs and DC motors are depicted in Figs. 7 and 8, respectively. From these two figures, one can see that, for these two cases, the required forces for the square type HMMs are larger than that of the cylinder type HMMs. Also, one can see that the DC driven forces are relatively small for these two cases, especially for case 2. This indicates that the DC driven force is not a main concern in optimization of the structure design of the HMM.

From these two figures, one can conclude that the mechanism generates a unique decoupled dynamic interaction between the micro motion (a large driven force by PZT) and the macro motion (a small driven force by DC motor) as the PZT forces (about 200 N) do not transfer to the DC motors (less than 10 N). That unique feature is completely different from the traditional serial macro/micro manipulator where the micro driving force will directly transfer to the macro driver.

Fig. 9 shows the maximum stresses of the HMMs under the macro motion. As mentioned before, the maximum stress occurs on the surface of the spatial flexural hinge and is larger about a factor of 10 than the maximum stress under the micro motion situation. From Fig. 9, one can see that the cylinder type HMM has a smaller stress than the square type HMM under the same driving condition. A big difference can be found for case 2 especially. Also, the maximum stress sustained by the spatial flexural hinge is the main constraint in the macro and macro/micro orientation movement for the HMMs, but the maximum stress is very small compared to the young modules of the material (<1%).

In summary, comparing these two type HMMs, it is shown that the cylinder type HMM has promising characteristics such as large orientation motion, low required driving force, and small stress in...
Table 3
Macro orientation results of HMMs (deg).

<table>
<thead>
<tr>
<th>DC input (mm)</th>
<th>Case 1</th>
<th>Case 2</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>One PZT</td>
<td>Two PZTs</td>
</tr>
<tr>
<td></td>
<td>Square type</td>
<td>Cylinder type</td>
</tr>
<tr>
<td>0.5</td>
<td>−0.51</td>
<td>−0.59</td>
</tr>
<tr>
<td>1</td>
<td>−1.02</td>
<td>−1.18</td>
</tr>
<tr>
<td>1.5</td>
<td>−1.53</td>
<td>−1.78</td>
</tr>
<tr>
<td>2</td>
<td>−2.03</td>
<td>−2.37</td>
</tr>
<tr>
<td>2.5</td>
<td>−2.54</td>
<td>−2.96</td>
</tr>
</tbody>
</table>

Table 4
Macro–micro orientation results of HMMs (deg).

<table>
<thead>
<tr>
<th>Hybrid inputs (mm)</th>
<th>Case 1</th>
<th>Case 2</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>One leg</td>
<td>Two legs</td>
</tr>
<tr>
<td></td>
<td>Square type</td>
<td>Cylinder type</td>
</tr>
<tr>
<td>0.501</td>
<td>−0.61</td>
<td>0.61</td>
</tr>
<tr>
<td>1.0025</td>
<td>−1.23</td>
<td>1.23</td>
</tr>
<tr>
<td>1.5005</td>
<td>−1.87</td>
<td>1.87</td>
</tr>
<tr>
<td>2.0075</td>
<td>−2.50</td>
<td>2.50</td>
</tr>
<tr>
<td>2.5010</td>
<td>−3.14</td>
<td>3.14</td>
</tr>
</tbody>
</table>

Fig. 7. PZT driven forces for hybrid motion.

Fig. 8. DC driven forces for hybrid motion.
the flexural hinge. Those features will be considered in the following optimal design of the HMM.

### 3.3. Dynamic analysis

Natural frequencies and mode shapes are important features in the design of a mechanical structure especially for a compliant mechanism under dynamic loading conditions. Dynamic analysis is also performed on the HMMs for these two cases. Table 5 presents natural frequencies of the first four modes for two types of HMMs under two different configurations. From this table, it can be seen that the designed HMMs have relative good dynamic behaviors (larger than 500 Hz). Also, it is clearly shown that increasing the section size of the HMM will increase the natural frequency of the system [19]. It should be mentioned that the first two modes are associated with the 3-axis spherical flexural hinges, while the next two modes are associated with the flexural hinges of the CMA. This means the stiffness of the CMA is larger than that of the spherical flexural hinge. It should be noted that the selected PZT actuator has a very high resonant frequency (above 100 kHz). Therefore, the integration of PZT to the complaint HMM (with 500 Hz) will not degrade the system dynamic performance.

### 4. Optimal design of HMM

In the past decades, structural optimization has been extensively explored and successfully applied to optimize structures and mechanisms in many practical engineering designs with the use of FEA. In the previous section, two types of spatial flexural hinges were studied to build the HMM and some primary conclusions were obtained. The purpose of an optimization process for the HMM is to obtain optimal design parameters associated with the HMM in order to achieve the maximum orientation while taking into account of the maximum stress occurred in the flexural hinges.

The Taguchi method [14], one of the optimal methods, has good reappearance of experiments concerned only with the main effects of design parameters. In principle, the Taguchi’s design of experiments is used to get information such as main effects and interaction effects of design parameters from a minimum number of experiments. The Taguchi method for quality engineering provides the capability of solving the optimal design problem of the HMM under limited experiment conditions. Compared with other experimental design methods, such as full factorial designs, fractional factorial designs, the Taguchi method is more efficient due to a smaller number of experiments needed. The Taguchi method consists of a set of experiments using the orthogonal array (OA) where settings of design parameters are changed at different levels.

#### 4.1. Design of experiment using orthogonal array

##### 4.1.1. Signal to noise ratio

In the field of communication engineering, a quantity called the signal to noise ratio (SNR) has been used as the quality characteristic of the choice. Taguchi introduced this concept into the design of experiments. He suggested the SNR as a measure of the mean squared deviation (MSD) in the performance. In our study, the larger-the-better SNR is used and defined as

\[
\text{SNR} = -10 \log_{10}(\text{MSD}) = -10 \log_{10}\left(\frac{1}{n} \sum_{i=1}^{n} \frac{1}{y_i^2}\right)
\]  
(15)

where \(n\) is the number of tests in a trial, and \(y_i\) is the value of system performance for the \(i\)th test in the trial. High SNRs are pursued in the Taguchi experiments.

For the HMM optimization, the system performance \(y\) is defined as

\[
y = \frac{|f|}{\sigma}
\]  
(16)
where $\beta$ (deg) is the orientation angle of the platform, and $\sigma$ (GP) is the first maximum stress of the HMM for the macro motion or the hybrid motion as the maximum stress for a micro motion is much smaller than that of the macro motion. The definition of this performance reflects the ideal goal, that is the HMM has a large orientation angle yet a small stress.

4.1.3. Experiment design of HMM

Structural optimization can be carried out in continuous or discrete design space. In this paper, an optimization process using the Taguchi method based on orthogonal arrays is conducted for designing the HMM in discrete space. The OA is selected on a discrete design space and levels are chosen from pre-defined values. Matrix experiments with the OA are carried out. Previous analysis (Table 1) has shown that 11 design parameters are the major contributors to the performance of the HMM. In the optimal design process, these 11 parameters or factors are set in three different levels based on the application of cell manipulation and the values are listed in Table 6. For the present study, the $L_{27}$ OA with 11 factors is chosen. Table 7 shows the selected parameter levels in each experiment in the OA table where 27 rows represent 27 different experiments driven by two legs. It demonstrates that the results of the hybrid motion are the combination of the micro motion and the macro motion. Fig. 11 presents the PZT driving force and maximum stress of HMM for different motions. It shows motion is the combination of the PZT displacement and the movement of the DC motor. After the FEA simulations, the characteristics of each combination of the design parameters are transformed into SNR according to Eqs. (15) and (16). The following section will show the analysis results.

4.2. Data analysis and optimal setting

According to the combination of the design parameters in the selected OA Table 7, 27 different finite element models of the cylinder type HMM are built and the static and dynamic analysis using ANSYS are performed to obtain the orientation angle, the PZT forces, and the maximum stress for each experiment. Fig. 10 shows the orientation angles of the micro, macro, and hybrid motions of 27 experiments driven by two legs. It demonstrates that the results of the hybrid motion are the combination of the micro motion and the macro motion. Fig. 11 presents the PZT driving force and maximum stress of HMM for different motions. It shows

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Table 7

$L_{27}$ orthogonal array experiments.

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Fig. 10. Orientation angles for different motions.
that the PZT driving forces for micro and hybrid motions are almost the same for each experiment, and the maximum stresses for macro and hybrid motions are very close. These results demonstrate the decoupling feature of the designed HMM. In the optimal process, three objective functions are defined. They are (1) the maximum micro orientation driven by the PZTs, (2) the maximum macro motion with the minimum stress driven by the DC motors, and (3) the maximum macro–micro orientation with the minimum stress under the hybrid motion.

For the optimal design of the micro orientation, the predicted data of the orientation angle $\beta$ are used to find the optimal design parameters using ANOM based on the SNR information. Fig. 12 shows the effect of each design parameter on the SNR. From this figure, it can be seen that, for the micro motion, parameter $t$ (the thickness of the flexural hinge on the CMA) is the most significant design parameter for the micro orientation. The smaller the parameter $t$, the bigger the orientation angle $\beta$. Also, the parameters $h$, $l$, and $L_4$ are important design parameters. On the other hand, the parameters $R_b$, $R_i$ (that are associated with the input flexural hinge of the CMA), $L_3$, $F_s$, and $R_s$ (that are associated with the spatial flexural hinge) have little effect on the micro orientation.

Fig. 13 shows the effects of the design parameters on the SNRs for the hybrid motion of the HMM based on the Taguchi method. One can see that the most significant design parameter on the system performance is parameter $L_s$ (the length of the spatial flexural hinge). The longer the spatial flexural hinge, the better the system performance. Other two design parameters, $F_s$ and $R_s$ (that are associated with the spatial flexural hinge), are important to the system performance as well. It should be noticed that the effect of design parameters on the macro motion is the same as the hybrid motion shown in Fig. 13.

From Figs. 12 and 13, the optimal levels of the design parameters for each optimal function can be found according to the largest SNR principle, and they are listed in Table 8.

Running confirmation experiments is necessary to compare the optimal results with the individual experimental results in order to demonstrate the effectiveness of the optimization process. Fig. 14 shows the 27 experiments and the optimal parameter experiments for the micro motion, the macro motion, and the hybrid motion, respectively. In this figure, the 27 experimental results are obtained from ANSYS using the listed design parameters in Table 7, and the optimal results are obtained using the optimal level design parameters listed in Table 8. From this figure, it can be seen that the optimal values (ignore the sign) are higher than that of every experimental result for each motion. As the optimum level for macro motion and hybrid motion are the same in Table 8, it implies that the macro motion dominates the objective function. Therefore, it is anticipated that Fig. 14b presenting the macro motion and Fig. 14c showing the hybrid motion are very similar.

5. Concluding remarks

In this paper, the design, modeling, and optimization of a HMM with 3-DOFs were presented. A unique feature of the developed HMM is the elimination of coupling interaction between the macro motion and the micro motion by proper mechanical design. For the micro motion of the designed HMM, a new structure of a compliant mechanical amplifier was used to build a leg for the HMM as the micro motion part to get a large micro motion with high accuracy and high natural frequencies. The HMM was discussed in aspects of design principle, kinematics design and analysis, prediction of the orientation movement based on two types of spatial flexural hinges. It was shown that the cylinder type HMM is promising in terms of a large orientation and small stress on the flexural hinge. After that, optimization of the HMM was performed for three objective functions using the Taguchi method. Some important design parameters have been identified, and the optimal levels for the design parameters have been determined as well.

It is concluded that, for the optimal design, the design parameters associated with the beam flexural hinge (t, l, h, and $L_4$) have large impacts to the orientation of the micro motion, and the design parameters related to the spherical flexural hinge ($L_3$, $F_s$, and $R_s$) have significant effect on the orientation and the maximum stress of the HMM for the macro motion and the hybrid motion as well. Verification tests are fulfilled to confirm the effectiveness of the optimal design parameters. The design concept of a parallel architecture for the HMM can certainly achieve a better performance.

Some possible future work may include a prototype verification of the designed HMM, motion planning of the macro/micro movement, and control system design and implementation of the HMM.

---

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References


