Contour Tracking Control for Multi-DOF Robotic Manipulators

P.R. Ouyang, Member, IEEE, V. Pano, and J. Acob

Abstract—In this paper, a new position domain PD control is proposed to control a multi-DOF nonlinear robotic manipulator for the purpose of improving contour tracking performance. To develop this new position domain control system, a robotic manipulator is treated as a master-slave system where the master motion is used as an independent reference through equidistantly sampling, while slave motions are described as functions of the master motion according to contour tracking requirements. A position domain dynamic model of the robotic manipulator is developed by transforming the original dynamic model developed in time domain to position domain. One benefit of this developed position domain PD control is that there is no tracking error for the master motion and only the tracking error of slave motions will affect the contour tracking performance. Stability analysis is conducted for the proposed position domain control, and it is successfully verified the good contour tracking performance through simulation study.

I. INTRODUCTION

Robotic manipulators possess complex and nonlinear coupled dynamics, and their control has been an intensive research topic during the past several decades. To achieve high performance requirements, many control systems have been developed [1-3]. Among them, PD/PID control is the most popular one applied in industrial robotic systems due to its simplicity and easy implementation. Theoretically, PD/PID control can asymptotically stabilize robotic systems in the sense of Lyapunov, and obtain acceptable tracking performances [3-7]. However, PD control is applied in decoupled individual joints and is based on tracking errors in a feedback loop. This decoupled control may provide good tracking performance for simple trajectory tracking defined at joint level. But for high precision contour tracking, as a motion of the end-effector is defined through synchronized movements of all active joints, PD control in individual joints may be insufficient for contour tracking. Developing a new control system for high precision contour tracking of a robotic system is the main motivation and starting point of this research.

Contour tracking control for a robotic manipulator is to control the motion of an end-effector following a desired contour effectively and precisely. A good tracking performance for each individual joint does not guarantee small contour errors for a robotic manipulator with multiple links connected serially. Furthermore, poor synchronization of relevant motion joints results in deteriorated dimensional accuracy of contour tracking [8-11]. For robotic manipulators, motion synchronization is very important when two or more joints have to move in a cooperative way [13-15] in order to obtain a good contour tracking of the end-effector. Motion synchronization has been studied in recent years for contour tracking in the manufacturing industry. To improve the contour tracking performance, a so-called cross-coupling control (CCC) was developed by Koren [8]. It is proven that CCC is an efficient motion control that can reduce tracking error for specific contours. The main idea of CCC is to achieve position synchronization of two motion axes.

Although CCC has been used in multi-axis motions for manufacturing [8, 11, 12], robotic manipulation at joint level [13-15], and aerospace operation [16], this method still has some limitations. One of them is the complexity involved in determining coupling gains [12]. Any inaccuracy in the determination of coupling gains is amplified to the control signal and results in a decreased contour performance. Furthermore, the majority of CCC is applied in CNC machining, and it was not widely used in the control of robotic manipulators that are quite different from machine tool in terms of complexities, objectives, requirements, and operating conditions [9].

Many studies show that a control system should synchronize joints/axes motions to achieve a good desired contouring performance. Obviously, the decoupled PD control has its limitations for this purpose. The classical master-slave control is a well known scheme where the command of a slave motion originates from the master motion in a synchronized way. In general, a master-slave control can achieve acceptable synchronization performance [17, 18].

In our previous research [19], a PD position domain control system is applied to 2-DOF linear translation motion manipulators. The purpose of this research is to extend the position domain control to nonlinear multi-DOF robotic manipulators. The main objective of position domain control is to improve the contour tracking performance through a simple and easy way. In this paper, the master-slave synchronization principle is adopted to develop the new position domain PD controller for contour tracking of robotic manipulators.

II. RELATIVE DERIVATIVE AND DYNAMIC MODEL IN POSITION DOMAIN

A. Relative Derivative

For a serial robotic manipulator with n-DOF, we assume the motion associated with joint 1 is a master motion and the motions associated with all other joints are slave motions. To develop position domain control, we need obtain the relationship between the master motion ($q_1$) and the slave motion ($q_i$). A relative derivative concept is introduced to define the relationship of master-slave motions. Similar with
the definition of a partial derivative, we define the relative derivative of joint \( i \) with respect to joint 1 as follows:

\[
q'_i = \frac{dq_i}{dq_1} \quad \text{for } i = 2, \ldots, n
\]  

(1)

From Eq. (1), one can see that \( q'_i \) is an angular speed ratio between the slave motion and the master motion, or it describes a synchronized motional relationship between these two motions. The relative derivative is called relative position velocity of joint \( i \) respect to joint 1.

Similarly, we define the second relative derivative or relative position acceleration as follows:

\[
q''_i = \frac{d^2q_i}{dq_1^2} \quad \text{for } i = 2, \ldots, n
\]  

(2)

From Eq. (1), we have:

\[
\dot{q}_i = q'_i \dot{q}_1
\]  

(3)

Differentiating Eq. (3) again, we get:

\[
\ddot{q}_i = q''_i \dot{q}_1^2 + q'_i \ddot{q}_1
\]  

(4)

Eqs.(3-4) represent the relationships between the absolute motions and the relative motions. Eq. (3) builds the relationship between the absolute velocity in time domain and the relative derivative in position domain, while Eq. (4) forms the relationship between the absolute acceleration and the relative acceleration. These two equations are used to develop the dynamic model in position domain.

\[ B. \ \text{Dynamic Model in Position Domain} \]

An n-DOF serial robotic manipulator with revolute joints has the following dynamic model [1-3]:

\[
M(q)\ddot{q}(t) + C(q, \dot{q}) \dot{q}(t) + G(q) + F(t, q, \dot{q}) = \tau(t)
\]  

(5)

To facilitate the discussion of the proposed position domain PD control, joint 1 was assumed as a master motion and all other joints are the slave motions. Then we can write the dynamic model in a sub-matrix form that includes a master motion and slave motions.

\[
\begin{bmatrix}
M_{11} & M_{1s} \\
M_{s1} & M_{ss}
\end{bmatrix}
\begin{bmatrix}
\ddot{q}_1 \\
\ddot{q}_s
\end{bmatrix}
+\begin{bmatrix}
C_{11} & C_{1s} \\
C_{s1} & C_{ss}
\end{bmatrix}
\begin{bmatrix}
\dot{q}_1 \\
\dot{q}_s
\end{bmatrix}
+\begin{bmatrix}
G_1 \\
G_s
\end{bmatrix} + \begin{bmatrix}
F_1 \\
F_s
\end{bmatrix} = \begin{bmatrix}
\tau_1 \\
\tau_s
\end{bmatrix}
\]  

(6)

The dynamic model in Eq.(6) is presented as a combination of a master motion and slave motions, where subscript 1 means link 1 or the master motion, and subscript \( s \) means all other links or the slave motions. As link 1 is chosen as the master motion link, \( q_1 \) will be the reference position (joint angle). Assuming \( q_1 \) is used as the reference for a defined contour tracking, the dynamic model for link 2 to link \( n \) or the slave motions can be rewritten as a function of the reference \( q_1 \) in position domain through an mathemetic transformation from time domain \( (t) \) to position domain \( (q) \).

Submitting Eqs. (3-4) into Eq. (6) of the slave motions (the dynamics of joint 2 to joint \( n \)), a dynamic model for the slave motions in position domain can be derived as:

\[
\dot{q}_i = q'_i \dot{q}_1, \quad \ddot{q}_i = q''_i \dot{q}_1^2 + q'_i \ddot{q}_1
\]  

[Remark 1]: Eq. (7) is the derived dynamic model represented in position domain through a transformation from time domain to position domain by assuming \( q_1 \) as the reference. As \( q_1 \) is assumed monotonically increasing or decreasing for each segment in position domain and \( t \) is monotonically increasing in time domain. Therefore, there is a one-to-one relationship between \( t \) and \( q_1 \), so \( q_1 \) and \( t \) are isomorphic [20]. It implies that any property which is preserved by the robotic manipulator in time domain in Eq. (5) is inherited in position domain in Eq. (7). It means that such a transformation doesn’t change the characteristics of the robotic manipulator, such as the stability of the system.

[Remark 2]: Eq. (7) also represents a general synchronized dynamic model where the slave motion is following the master motion according to the contour requirement. As the master joint \( q_1 \) is a reference and a base, a high precision measurement for joint \( q_1 \) is highly desired in order to achieve high precise contour tracking performance. In other words, a high space sampling frequency is very helpful to ensure good contour tracking of the slave motions, and such a requirement can be easily satisfied by using a high resolution encoder to measure the angular position.

III. POSITION DOMAIN PD CONTROL AND PROPERTY

A. Position Domain PD Control

Similar to time domain PD control, a position domain PD control law is proposed to control the slave motions of a multi-DOF robotic manipulator described in Eq. (7) as follows:

\[
\tau_i(q_i) = K_{pe}e_i(q_i) + K_{de}e'_i(q_i)
\]  

(8)

Where \( K_{pe} \) and \( K_{de} \) are diagonal proportional gain matrix and derivative gain matrix, respectively. The tracking position error and the relative derivative error in position domain are defined as:

\[
e_i(q_i) = q_{sd}(q_i) - q_i(q_i) = \\
\begin{bmatrix}
q_{sd}(q_i) - q_1(q_1) \\
q_{sd}(q_i) - q_2(q_2) \\
q_{sd}(q_i) - q_n(q_n)
\end{bmatrix}
\]  

(9)

\[
e'_i(q_i) = q'_{sd}(q_i) - q'_i(q_i) = \\
\begin{bmatrix}
q'_{sd}(q_i) - q'_1(q_1) \\
q'_{sd}(q_i) - q'_2(q_2) \\
q'_{sd}(q_i) - q'_n(q_n)
\end{bmatrix}
\]  

[Remark 3]: The proposed PD control can be viewed as cooperative control between the master motion and the slave motions. Also, position domain PD control can be viewed as a more general synchronized position control where the slave motion of joint \( q_i \) follows the motion (both position and velocity) of master joint according to the contour requirements. Therefore, the developed position domain PD control is a synchronous position and velocity control that
keeps the motion relationship as accurate as possible between
the master motion and the slave motions.

B. Properties and Assumptions of the Dynamic Model

In this paper, a list of properties of a rigid robotic
manipulator [2-5] associated with Eq. (5) and Eq. (7) is used
in the stability analysis and is described as follows:

P1: the inertia matrix \( M(q) \) is symmetric positive
definite. It is easy to prove that the sub-matrix \( M_s \) is also
symmetric positive definite.

P2: the matrix \( M(q) - 2C(q, \dot{q}) \) is a skew symmetric
matrix, so does \( \dot{M}_s(q) - 2\dot{C}_s(q, \dot{q}) \)

P3: the inertia and centrifugal-Coriolis matrices satisfy
the following relationship.

\[
\dot{M}_s(q) = C_s(q, \dot{q}) + C'_s(q, \dot{q}) 
\] (10)

P4: \( M(q), C(q, \dot{q}), G(q), \) and \( F(t, q, \dot{q}) \) are bounded.

Also the following notations are introduced.

\[ \lambda_s(M) \text{ and } \lambda_{st}(M) \] represent the smallest and the
largest eigenvalues of a positive define matrix \( M \). If a square
matrix \( M \) is positive definite, then it is denoted as \( M > 0 \); If
a square matrix \( M - N \) is positive definite, then it is denoted as
\( M - N > 0 \).

For positive definite matrices, the following properties
[20] will be used in this paper:

P5: If \( M > 0 \), then \( M^{-1} > 0 \).

P6: If \( M \geq N > 0 \), then \( N^{-1} \geq M^{-1} > 0 \).

P7: If \( M > 0 \) and \( \lambda > 0 \) is a real number, then \( \lambda M > 0 \).

P8: If \( M > 0 \) and \( N > 0 \), then \( M + N > 0 \), \( MNM > 0 \),
and \( MNM > 0 \).

Finally, the following reasonable assumptions are used in
this paper.

A1: The master motion \( q_i \) is a continuous function with
the second-order derivative for \( q_i \in [q_{iL}, q_{iU}] \).

A2: The velocity \( \dot{q}_i \) and acceleration \( \ddot{q}_i \) of the master
motion are bounded in the desired contour trajectory region.

A3: The desired contour trajectory \( q_d(q) \) is the second–
order continuous.

IV. STABILITY ANALYSIS

A. Lemma

Assume a matrix \( Q \) is a symmetric matrix expressed as:

\[
Q = \begin{bmatrix}
A & B \\
B^T & C
\end{bmatrix}
\]

Let \( S \) be the Schur complement [20] of matrix \( A \) in \( Q \),
that is:

\[
S = C - B^T A^{-1} B 
\] (11)

Then the matrix \( Q \) is positive definite if and only if \( A \) and
\( S \) are both positive definite [20]. It means that if \( A \succ 0 \) and
\( S \succ 0 \), then \( Q \succ 0 \).

B. Theorem for Position Domain PD Control

Theorem: For a rigid robotic manipulator described in
position domain of Eq. (7), if the position domain PD
control law in Eq. (8) is applied to control a contour tracking
of the robotic manipulator, and the following conditions in
Eq. (12) are satisfied, then the controlled robotic
manipulator is globally asymptotically stable for contour
tracking.

\[
\begin{align*}
K \succ 0 \\
K_p \succ 0, \ K_m \succ 0 \\
K + \dot{q}_i C_{ss} - \ddot{q}_i M_s \succ 0 \\
\lambda_s(K_{ps}) > \dot{q}_i^2 \cdot \lambda_{st}(M_s) > 0 \\
\lambda_s(K_{ps}) > \frac{1}{2} \lambda_{st}(K + \dot{q}_i C_{ss} - \ddot{q}_i M_s)
\end{align*}
\] (12)

C. Stability Analysis

To prove the stability of the proposed position domain
PD control law, we first prove the following matrix \( Q \) is
symmetric positive definite.

\[
Q = \begin{bmatrix}
K_p & \dot{q}_i M_s \\
\dot{q}_i M_s & \ddot{q}_i M_s
\end{bmatrix} 
\] (13)

Proof:

PD control gains are symmetric diagonal matrices with
positive constant elements. From Eq. (11), we know that \( K_p \)
is symmetric positive definite matrix. From P1, we know
that \( M_s \) is symmetric positive definite, Therefore, the matrix
\( Q \) is a symmetric matrix.

From conditions (11), we have:

\[
K_p - \ddot{q}_i K_{ps}^{-1} \succ 0 
\] (14)

As \( K_p \succ 0 \) and \( M_s \succ 0 \), according to properties P5 - P7,
we can prove:

\[
M_{s}^{-1} - \ddot{q}_i K_{ps}^{-1} \succ 0 
\] (15)

Furthermore, based on (15) and \( M_s \succ 0 \), according to the
property P8, we have:

\[
\dot{q}_i^2 (M_s (M_{s}^{-1} - \ddot{q}_i K_{ps}^{-1}) M_s) > 0 
\] (16)

According to the property P1 and reorganizing Eq. (16),
we have: \( S = \dot{q}_i M_s - (\dot{q}_i C_{ss} M_{s}^{-1} (\ddot{q}_i M_s)) > 0 \). According to the Preposition and Eq. (11), we prove that the matrix \( Q \) in
Eq. (13) is symmetric positive definite.

Q.E.D.

Using Eq. (9), the dynamic model in Eq. (7) can be re-
expressed in an error function format as follows:

\[
\begin{align*}
\rho &= \dot{q}_i^2 M_s q_d^* (q) + \dot{q}_i M_s q_{d, \dot{q}} (q) + M_s \ddot{q}_i + C_s \ddot{q}_i + G + f_s \\
\dot{q}_i M_s e^* (q) + (K_p + M_s \ddot{q}_i + C_s \ddot{q}_i) e' (q) + K_m e (q) &= \rho
\end{align*}
\] (17)

According to the properties and the assumptions, we have:
\[ \rho \leq \left\| \frac{\partial}{\partial q} \dot{M}_w \dot{q} + \frac{\partial}{\partial \dot{q}} \dot{M}_w \dot{q} + M_w \ddot{q} + C_w \ddot{q} + G_w + F \right\| \]

\[ \leq m_n \left\| \frac{\partial}{\partial q} \ddot{q} + \frac{\partial}{\partial \dot{q}} \ddot{q} + m_i \dddot{q} + c_i \| \| + \left\| \dot{\gamma} \right\| + \left\| \dot{\varepsilon} \right\| = \left\| \rho \right\| \] (18)

It means that the parameter \( \rho \) is bounded.

For the dynamic system in position domain described in Eq. (7), we define the following Lyapunov function:

\[ V(\epsilon(q), \dot{\epsilon}(q)) = \frac{1}{2} (\epsilon^T \epsilon + \epsilon^T \epsilon'(K + K_{\ast}) \epsilon) \]

\[ = \frac{1}{2} \begin{bmatrix} \epsilon_s^T & \epsilon_s' \end{bmatrix} \begin{bmatrix} K_{\nu} & M_{\nu} \dot{q}_i \\ M_{\nu} \dot{q}_i & M_{\nu} \dot{q}_i' \end{bmatrix} \begin{bmatrix} 
\epsilon_s \\
\epsilon_s' \end{bmatrix} + \frac{1}{2} \epsilon_s^T (K + K_{\ast}) \epsilon_s 
\] (19)

From the above discussion, we have that \( Q \) is a symmetric positive definite matrix. According to Eq. (11), we know that \( K + K_{\ast} \) is also a symmetric positive definite matrix. Therefore the Lyapunov function is a positive definite function:

\[ V(\epsilon(q), \dot{\epsilon}(q)) > 0 \] (20)

In position domain control, the reference angular position \( q_0 \) is an independent variable that has the similar meaning of \( t \) in time domain. \( \epsilon_s \) and \( \dot{\epsilon}_s \) are functions of the independent variable \( q_0 \). Therefore, the derivative of \( V \) is related to variable \( q_0 \) in this stability analysis.

From Eq. (20), the derivative of \( V \) along the contour tracking errors of the system is given by

\[ \frac{dV}{dq_0} = e_s^T (K_{\nu} + K_{\ast} + K + \dot{q}_i M_{\nu}) e_s + e_s^T \left( \dot{q}_i M_{\nu} + \frac{\dot{q}_i}{2} M_{\nu} \right) e_s' \]

\[ + \left( e_s^T + e_s'^T \right) \dot{q}_i^T M_{\nu} e_s^T \quad (21) \]

Considering properties P2 and P3, and applying Eqs. (10, 17,18) to Eq. (21), we obtain:

\[ \frac{dV}{dq_0} = -e_s^T K_{\nu} e_s - e_s^T \left( K_{\nu} + \left( \dot{q}_i - \dot{q}_i^T \right) M_{\nu} \right) e_s' \]

\[ + e_s^T \left( K + \dot{q}_i C_{\nu}^T - \dot{q}_i M_{\nu} \right) e_s' + \left( e_s^T + e_s'^T \right) \rho \quad (22) \]

As \( K + \dot{q}_i C_{\nu}^T - \dot{q}_i M_{\nu} \) is positive definite from Eq. (11), we have:

\[ e_s^T \left( K + \dot{q}_i C_{\nu}^T - \dot{q}_i M_{\nu} \right) e_s' \leq \frac{\lambda_{\nu}}{2} \left( K + \dot{q}_i C_{\nu}^T - \dot{q}_i M_{\nu} \right) \left[ e_s^T e_s + e_s'^T e_s' \right] \quad (23) \]

Applying Eq. (23) to Eq. (22), we get:

\[ \frac{dV}{dq_0} \leq \left( \left\| \varepsilon \right\| + \left\| \dot{\varepsilon} \right\| \right) \left\| \rho \right\| - e_s^T K_{\nu} \left( K + \dot{q}_i C_{\nu}^T - \dot{q}_i M_{\nu} \right) e_s' \]

\[ - e_s^T \left( \dot{q}_i M_{\nu} \right) \left( K + \dot{q}_i C_{\nu}^T - \dot{q}_i M_{\nu} \right) e_s' \]

Assume:

\[ \rho = \lambda_{\nu} \left( K + \dot{q}_i C_{\nu}^T - \dot{q}_i M_{\nu} \right) \]

\[ \rho = \lambda_{\nu} \left( K + \dot{q}_i C_{\nu}^T - \dot{q}_i M_{\nu} \right) I \]

Then Eq. (24) can be rewritten as:

\[ \frac{dV}{dq_0} \leq -\rho_s \| \varepsilon \| - \rho_s \| \dot{\varepsilon} \| \quad (25) \]

Finally, we get:

\[ \frac{dV}{dq_0} \leq -\rho_s \| \varepsilon \| + \left( \frac{1}{\rho_s} + \frac{1}{\rho_s} \right) \| \rho \| \quad (26) \]

Based on the Lyapunov theorem, we conclude that the robotic manipulator controlled by the position domain PD control law is globally ultimately bounded. The bounded errors for both the tracking error and the relative derivative of the tracking error can be obtained as follows:

\[ \left\| \varepsilon \right\| \leq \frac{1}{\rho_s} + \frac{1}{\rho_s} \| \rho \| \quad (27) \]

V. SIMULATION STUDY AND VERIFICATION

In this section, we will present some simulation study to demonstrate the effectiveness of the proposed position domain PD control for different contour tracking. A 3-DOFs revolute joint serial robotic manipulator [21] shown in Fig. 1, with structural parameters listed in Table 1, is used as an example for contour tracking control experiments. The setup of this 3-DOF planar robotic manipulator is on the vertical plane.

![Fig. 1 Scheme of a 3-DOFs robotic manipulator.](image_url)

<table>
<thead>
<tr>
<th>Link</th>
<th>Mass ( m_i ) (kg)</th>
<th>Length ( l_i ) (m)</th>
<th>Center ( r_i ) (m)</th>
<th>Inertia ( I_i ) (kgm²)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.00</td>
<td>0.50</td>
<td>0.25</td>
<td>0.10</td>
</tr>
<tr>
<td>2</td>
<td>1.00</td>
<td>0.50</td>
<td>0.25</td>
<td>0.10</td>
</tr>
<tr>
<td>3</td>
<td>0.50</td>
<td>0.30</td>
<td>0.15</td>
<td>0.05</td>
</tr>
</tbody>
</table>

In this paper, a fifth-order polynomial model for path planning is adopted for the motion of the end-effector in...
time domain control. Path planning for the slave motion is realized coordinately from the contour requirement through the inverse kinematics analysis of the robotic manipulator.

In the following subsections, two different contour tracking tests are conducted to verify the proposed position domain PD control and demonstrate its potential of high contour tracking performance.

A. Linear Motion of the End-effector

First, the linear motion in the end-effector level is conducted using the proposed position domain PD control, and comparison with the time domain counterpart is performed to examine the effectiveness of the new control method. For all linear motion conducted in the simulations, the following control gains are selected

For time domain PD control:
\[ K_p = \text{diag}[3000, 3000, 2500], \quad K_d = \text{diag}[2000, 1750, 1250] \]

For position domain PD control of the slave motion:
\[ K_p = \text{diag}[3000, 2500], \quad K_d = \text{diag}[1750, 1250] \]

It is supposed that the end-effector of the 3-DOF robotic manipulator to follow a line segment starting from point A (0.50, 0.0) and stopping at point B (0.25, 1.0) in 4 Sec. with a fixed orientation of 60°. Inverse kinematic analysis is conducted to form the trajectories in the joint levels. PD control in time domain and position domain are used to control the three actuators.

Fig. 2 shows the tracking errors for time domain PD control and position domain PD control, respectively. From this figure, one can see that the tracking errors for both control methods are almost the same order with a relatively large error for joint 3 in position domain. But in the end-effector level, the potential of the proposed position domain PD control is clearly demonstrated for the improvement of contour tracking, as shown in Fig. 3. The maximum contour error for position domain PD control is about 0.4 mm, while the maximum error for time domain PD control is above 1.0 mm.

B. Full Circular Motion of the End-effector

In this simulation, the end-effector of the 3-DOF robotic manipulator is required to finish a full circular motion for 8 sec. A circular contour for the end-effector is defined with a radius of 0.6 mm at the center of (0.0, 0.1), and the orientation angle of the end-effector is set to be constant of 45°. The control gains for time domain and position domain PD control are set the same and shown as follows:

For time domain PD control:
\[ K_p = \text{diag}[3000, 3000, 3000], \quad K_d = \text{diag}[2000, 2000, 2000] \]

For position domain PD control of the slave motion:
\[ K_p = \text{diag}[3000, 3000], \quad K_d = \text{diag}[2000, 2000] \]

Fig. 5 shows the desired and real contours obtained by both PD control methods where the tracking errors are amplified by 20 times. It shows that, one the right side of the real contour, the position domain PD control is better than
the time domain PD control. The good contour performance controlled by position domain PD control is proved in Fig. 6 where the maximum contour errors are 0.8 mm and 1.0 mm for position domain PD control and time domain PD control, respectively.

![Tracking result in time domain](image1)

![Amplified tracking in position domain](image2)

**Fig. 5** The tracking performance resulted from both PD control. (with 20x amplification of tracking errors)

![Contour errors in time/position domain](image3)

**Fig. 6** Comparison of contour errors for circular motion.

VI. CONCLUSIONS

In this paper, position domain PD control is proposed as an alternative to time domain PD control. A position domain dynamic model for a multi-DOF serial robotic manipulator is developed by transforming the original dynamic equations from time domain to position domain. Contour tracking control is used for motion control of the end-effector of a multi DOF robotic manipulator following a desired contour effectively and precisely. The stability of position domain control is guaranteed by the Lyapunov method. Different types of motions in the end-effector level are used to test the effectiveness of the proposed position domain PD control.

It is shown that position domain PD control can obtain a better contour tracking performance compared to the counterpart in time domain. Further study and performance evaluation considering the external disturbance or noise are needed. Also, experimental verification for the proposed position domain PD control is a future work.

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REFERENCES