A Downtown On-Street Parking Model with Urban Truck Delivery Effects: A Case Study of Toronto’s Financial District

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EXECUTIVE SUMMARY

In this study we present an on-street parking model for downtowns in urban centers that incorporates the often-neglected parking demand of commercial vehicles. When parking is saturated, passenger cars often cruise until an open space is available. Commercial vehicles, on the other hand, are more likely to double-park near their destinations and occupy a travelling street lane.

We study the relation between commercial and passenger vehicles’ parking behaviours, and we show how overlooking a segment of road users and their travel behaviour is often a source of disruption to busy road networks.

The study presents an analytical economic-based model that evaluates the effects of different parking policies in urban centres with respect to network congestion, cruising, double-parking, and the travel behaviour of commercial and passenger-vehicles.

The presented model is able to distinguish between four types of travelers that make up the traffic composition of the streets in the downtown including vehicles cruising for parking and vehicles that occupy part of the street space by double-parking. And it is able to incorporate the congestion effect that every type contributes to the traffic. The model then provides tools for policy makers to optimize the trade-offs in parking spaces, pricing, and network congestion. To the best of our knowledge, this is the first parking equilibrium model that considers all these trade-offs.

The model can be easily customized to other downtown regions around the world to support similar policy recommendations.

We apply the model to a case study area that encompasses the Financial District in downtown Toronto to demonstrate the application of the model and how useful it could be in creating significant gains in social surplus (in maximizing the total benefit minus the total cost). We found that compared to a baseline scenario representative of the study area in Toronto in 2015, increasing parking fees from $4/hour to nearly $9/hour and assigning 3.4% of parking spaces to truck deliveries would eliminate cruising and truck double-parking, resulting in a social surplus gain of over $13,500/hr/mi².

The results of the case study indicate that it is necessary for policy makers to capture the effect of all road users including commercial delivery trucks and their parking behaviour. The current practice of overlooking the effect of this segment of road users and resorting to parking fines instead will inevitably result in devising less efficient policies when it is most needed to reduce congestion. In the case study we have demonstrated how the developed model captures all the segments of road users and optimizes the road space accordingly allowing the most efficient allocation of on-street parking and the optimum corresponding parking fees.
1. INTRODUCTION

As the rate of urbanization increases, societies struggle to develop policies to make the most efficient use of land to cope with congestion. Parking management is one such policy. Poorly implemented parking policies can lead to “cruising” for parking spaces, which can account for more than 30% of downtown traffic in some cases (Shoup, 2005)\(^1\). On the other hand, parking pricing strategies can be more effective than road pricing strategies because of a greater public acceptance. The effectiveness of parking policies can also be enhanced by such engineered technologies as real time information systems (e.g. Cao and Menendez, 2015)\(^2\) like SFpark.org or data-driven parking pricing (Qian and Rajagopal, 2013; Mackowski et al., 2015)\(^4\).

Researchers have developed analytical means of evaluating trade-offs in pricing, capacity, information technologies, and spatial-temporal allocation of parking spaces with respect to their welfare effects on cruising, traffic congestion, transit use, and activity patterns, among others. However, urban freight is largely neglected in these studies, despite the significant differences in freight parking use patterns from commuter patterns, the high demand for freight parking or loading/unloading, and the exacerbated effects that truck delivery inefficiencies have on multiple aspects of urban sustainability—congestion, safety, air quality, etc. (Chow et al., 2010; You et al., 2015). In a recent study of freight parking demand in New York City, Jaller et al. (2013) confirmed that parking policies often overlook urban freight.

Urban freight parking needs are inherently different from commuter parking. Unlike commuters, delivery trucks typically need spaces to temporarily park or to load or unload goods at destinations in the central business district. Trucks take up more space, require close proximity to destinations (Tipagornwong and Figliozzi, 2015), and require access routes to parking locations with greater turning radii. For example, parcel delivery services like FedEx, UPS, and Purolator accounted for more than $1.5M in parking fines in Toronto in 2006 (Haider, 2009). Jaller et al. (2013) highlight a list of example parking policies available to policy-makers: parking management systems, car-share

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provision, in-lieu fee, maximum parking standard, parking freeze, residential parking permits, transferable parking rights, variably priced parking, among others. These policies typically overlook freight or commercial vehicle parking demand.

In a focus group survey of thirteen industry sectors, Morris et al. (1998)\(^{10}\) identified parking as one of the key transportation barriers for freight mobility. Focus groups indicated congestion, inadequate docking space, inadequate curb space for commercial vehicles, and oppressive parking regulations as examples. Recommendations included off-peak deliveries, reducing passenger vehicle traffic, improving mass transit to reduce private passenger vehicles, creating “truck only” areas like the garment district in New York City, using integrated information systems, or introducing consolidation centres outside the city. While some strategies like off-peak deliveries have been studied further (e.g. Holguín-Veras et al., 2011)\(^{11}\), there are generally no analytical downtown parking models that consider freight delivery activities. The few efforts that do exist are either simulation-based (Nourinejad et al., 2014)\(^{12}\) or do not consider equilibrium interactions of truck deliveries and passenger parking (Tipagornwong and Figliozzi, 2015)\(^{8}\). As such, many of the recommendations or issues in urban freight and city logistics related to parking cannot be addressed.

We propose a downtown on-street parking equilibrium model that incorporates the effects of urban freight. The model generalizes a state-of-the-art on-street parking model (Arnott and Inci, 2006)\(^{13}\) to include effects of space allocation for truck deliveries, truck double parking, and consequences in traffic flow capacities. To the best of our knowledge, this is the first parking equilibrium model that considers all these trade-offs. We then apply the model to a case study of downtown Toronto to support first-best and second-best space allocation policies for parking spaces. The model can be easily customized to other downtown regions around the world to support similar policy recommendations.

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2. **Literature Review**

Analytical commuter parking models are relatively new compared to other transportation models. Some of the earliest models of note examined the dual nature of parking as a private and public good. Glazer and Niskanen (1992)\[^{14}\] noted that economists (e.g. Vickrey, 1954)\[^{15}\] generally assumed curbside parking to be a private good to justify marginal cost pricing. On the contrary, the authors demonstrated that insufficient parking spaces lead to cruising behaviour, which results in increased costs for both travelers looking for parking as well as in-transit travelers. When the roadway is suboptimally priced or free, there should be a positive lump sum parking fee that covers that cost.

Another feature of the dual nature of parking observed by Arnott et al. (1991)\[^{16}\] and Anderson and de Palma (2004)\[^{17}\] is that the pricing by a market of private operators is both monopolistic and competitive. Each operator sets the price as profit-maximizing due to the all-or-nothing demand for a single space (this behaviour has been empirically confirmed by Kobus et al., (2013)\[^{18}\], but is competitive with other parking spaces for a user. Because operators may ignore the costs they impose on cruising, it is possible that the competitive pricing may result in welfare reduction relative to no pricing at all.

Arnott et al. (1991)\[^{16}\] used Vickrey’s (1969)\[^{19}\] bottleneck congestion model to derive insights on the spatial and temporal nature of parking pricing. When parking is free, the authors showed how driver behaviour to naturally park “outwards”—occupy spots in order of decreasing accessibility—leads to increased inefficiencies. Time-varying road pricing may eliminate queuing and reduce schedule delay costs, but distance-based parking pricing is needed to induce a more efficient “inward” parking behaviour. They concluded that it is easier to implement an efficient parking fee policy than efficient road tolling policy. Their bottleneck model of parking has been extended by Zhang, Huang, and Zhang (2008)\[^{20}\] to consider both morning and evening commutes, by Zhang, Yang, and Huang (2011)\[^{21}\] to investigate the efficiency of parking permits, by Qian, Xiao, and Zhang (2012)\[^{22}\] to examine parking clusters, and by Yang et al. (2013)\[^{23}\] to add capacity constraints and parking

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reservations. Fosgerau and de Palma (2013)\textsuperscript{24} studied the effects of early bird specials with time-varying parking pricing. While Lam et al.’s (2006)\textsuperscript{25} work is not directly a bottleneck parking model, they considered departure time choice at a network level using variational inequalities. The model requires route enumeration, which makes it difficult to apply to large-scale study areas.

Arnott and Rowse (1999)\textsuperscript{26} used a circular city structure to analyze the randomness of parking availability and cruising to examine dynamic parking pricing and justify parking information systems. The model structure resulted in non-unique equilibria, however, and required a number of assumptions including ignoring traffic congestion. Anderson and de Palma (2004)\textsuperscript{17} incorporated cruising in a simpler model to arrive at several major conclusions. First, the socially optimal parking configuration is independent of the cost of cruising. However, the equilibria of both unpriced parking and privately operated parking have smaller optimal parking spans as cruising costs increase, though the price of parking is always better off than the unpriced parking.

Arnott and Inci (2006)\textsuperscript{13} first introduced a parking equilibrium model (denoted “AI06 model”) with traffic flow behaviour to explicitly measure cruising effects. They found that regardless of the curbside parking capacity, it is efficient to price the spots to the point where cruising can be eliminated without parking becoming unsaturated. On the other hand, if pricing is fixed, then it is second-best optimal to increase the number of curbside spaces until cruising is eliminated without parking becoming unsaturated.

In more recent years, research on parking has shifted to interactions between multiple decision-makers. Calthrop and Proost (2006)\textsuperscript{27} studied curbside parking in the presence of off-street parking using a Stackelberg game with a single garage operator as a follower. Arnott (2006)\textsuperscript{28} extended his earlier traffic flow explicit parking model to include spatial competition between parking garages and curbside parking under a public authority. The study includes a variant model that accounts for mass transit, allowing policy-makers to evaluate trade-offs between system-wide transit designs and parking policies. Several conclusions were made: competition between parking operators determines the full price of parking; cruising costs adjust the curbside parking pricing to match the garage parking; increasing saturated curbside parking prices reduces cruising and traffic congestion; mass transit can significantly affect second-best parking policy, which can be exploited by considering maximum garage parking standards (done so in Boston, New York, and San Francisco). Arnott and Rowse (2009)\textsuperscript{29} illustrated the model with a detailed numerical example.


\textsuperscript{28} Arnott, R., 2006. Spatial competition between parking garages and downtown parking policy. Transport Policy 13, 458-469.

The two leading analytical model structures in the literature appear to be the AI06 model and the bottleneck parking model, each with their own benefits and limitations. Neither class of models currently deals with truck delivery behaviour. As a consequence, we cannot evaluate the effects of congestion impacts between trucks, personal in-transit vehicles, cruising vehicles, and double-parking vehicles; curbside space capacity for trucks; time windows for deliveries; integrated information systems or advanced connected truck technologies; or consolidation centers. In this study, the AI06 model is generalized to include truck traffic and delivery behaviour.
3. The Model

The model is developed based on Arnott and Inci (2006)\textsuperscript{13} downtown parking and traffic congestion model with a key expansion made to consider commercial vehicles parking and to provide a tool for policy makers to control the double-parking behaviour of CVs along with the cruising behaviour of passenger cars. A key benefit in building on AI06 is that it considers elastic travel demand for passenger cars. This means the demand is price sensitive, as the trip price increases, either in terms of money or in terms of time, the traffic demand declines, and vice versa. In addition to controlling the cruising and double-parking behaviours of travelers, the elastic demand allows to some degree an overall regulation of the travel demand by changing the total trip price. This makes parking policies able to share some of the traffic regulation effects of road tolls.

In the next four subsections, we describe progressively; each subsection discusses part of the model.

3.1 Assumptions and downtown setting description

Before proceeding with describing the model, it is important to mention a few notes to help distinguish between passenger cars and commercial vehicles as they are intended in this study. First the size of commercial vehicles and their maneuvering capabilities are quite different from passenger cars, and it is sensible that we distinguish between the typical curbside parking spaces available for passenger cars and those required for commercial vehicles. In this study we consider only light commercial vehicles (similar to those used by express courier delivery companies) for which it is more likely applicable for curbside parking than loading/unloading docks that are predominantly meant for larger vehicles with different types of cargo and with much longer parking periods at the destination.

With this perspective in mind, light commercial vehicles are still different from passenger cars and would require special parking spaces; therefore we assume that for a specific curbside parking space it would be necessary to assign it to either one type of these vehicles. And to distinguish between both parking spaces we denote the stock of passenger car parking spaces per unit area as $P_p$ and the stock of commercial vehicles parking spaces per unit area as $P_c$.

Another important note is the distinction between passenger car and commercial vehicles behaviours when curbside parking spaces are fully occupied. In such case, passenger car drivers may have to cruise around the area until an available space is found. This particularly occurs when curbside parking is underpriced, as it makes economic sense to search for cheap parking spaces.

Commercial vehicles on the other hand do not cruise for parking. Due to the high value of time attached to their trip, if no parking spaces were immediately available in close vicinity to their destination, commercial vehicles will resort to double-parking as the cheapest choice in hand. This major difference is incorporated in the model.
As we make clear these notes, we proceed with describing the model’s assumptions and variables. The model assumes a downtown area that features a grid street network with city blocks of side length $b$ and street width equal to $w$ and where parking is provided uniformly on-street. (Figure 2) below shows an initial set of variables used to describe the different types of the travelling vehicles on downtown streets.

**Figure 2**
Set of variables describing travelling vehicles

<table>
<thead>
<tr>
<th>Notation</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$D_p$</td>
<td>Passenger car trip demand per unit time-area (veh/hr-mi$^2$)</td>
</tr>
<tr>
<td>$D_c$</td>
<td>Commercial vehicles trip demand per unit time-area (veh/hr-mi$^2$)</td>
</tr>
<tr>
<td>$T_p$</td>
<td>Stock of in-transit passenger cars per unit area (veh/mi$^2$)</td>
</tr>
<tr>
<td>$C$</td>
<td>Stock of cruising passenger cars per unit area (veh/mi$^2$)</td>
</tr>
<tr>
<td>$T_c$</td>
<td>Stock of in-transit commercial vehicles per unit area-time (veh/mi$^2$)</td>
</tr>
<tr>
<td>$H$</td>
<td>Stock of double-parking commercial vehicles per unit area-time (veh/mi$^2$)</td>
</tr>
<tr>
<td>$P_p$</td>
<td>Parking spaces allocated to passenger cars per unit area (space/mi$^2$)</td>
</tr>
<tr>
<td>$P_c$</td>
<td>Parking spaces allocated to commercial vehicles per unit area (space/mi$^2$)</td>
</tr>
<tr>
<td>$\theta$</td>
<td>Ratio of a commercial vehicle parking space to that of a passenger car parking space.</td>
</tr>
<tr>
<td>$m_c$</td>
<td>Distance travelled by commercial vehicles in downtown before arriving to destination (mi)</td>
</tr>
<tr>
<td>$l_p$</td>
<td>Parking duration of passenger cars (hr)</td>
</tr>
<tr>
<td>$l_c$</td>
<td>Parking duration of commercial vehicles (hr)</td>
</tr>
<tr>
<td>$p_p$</td>
<td>Value of time of passenger cars ($/hr)$</td>
</tr>
<tr>
<td>$p_c$</td>
<td>Value of time of commercial vehicles ($/hr$)</td>
</tr>
</tbody>
</table>
For passenger cars, the demand for travel per unit area is $D_p$, and drivers are assumed to be homogenous with value of time equal to $\rho_p$. They must travel a distance $m_p$ through the downtown before arriving to destination. $T_p$ is the stock of passenger vehicles in-transit per unit area until they arrive to destination. Once there, if parking is available they park for a period of time $l_p$, otherwise, they cruise until a space is available. $C$ is the stock of cruising vehicles per unit area.

Commercial vehicles have different travel behaviour and therefore another set of variables $(D_c, \rho_c, l_c, m_c, T_c)$ are used to identify the above characteristics with the exception of cruising, which is only recognized for passenger cars. The stock of double-parking vehicles per unit area is denoted $H$, which is only considered for commercial vehicles.

With the introduction of these variables we are able to distinguish between four types of travelers that make up the traffic composition in the streets of the downtown. First, for passenger cars we have vehicles in-transit to destination $T_p$ and other vehicles cruising for parking $C$. Second, for commercial vehicles we have vehicles in-transit to destination $T_c$ and other vehicles $H$ that occupy part of the street space by double-parking. Finally, there are two other types of vehicles $P_p$ and $P_c$ that occupy a non-travelling part of the street space.

### 3.2 Travel congestion in the model

Travel is subject to flow congestion and in this section we aim to distinguish between the congestion effects that every type of travelling vehicles $(T_c, T_p, C, H)$ contribute to the traffic. The fundamental traffic flow relationships as expressed in Greenshield’s model are applied to determine the traffic state but with some modifications to account for travelers’ types discussed earlier.

Consider the set of variables in (Figure 3) which describe the traffic state.

<table>
<thead>
<tr>
<th>Notation</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$v$</td>
<td>Travel speed (mi/hr)</td>
</tr>
<tr>
<td>$v_0$</td>
<td>Free flow speed (mi/hr)</td>
</tr>
<tr>
<td>$t$</td>
<td>Travel time per unit distance (hr/mi)</td>
</tr>
<tr>
<td>$t_0$</td>
<td>The free flow travel time (hr/mi)</td>
</tr>
<tr>
<td>$k$</td>
<td>Density per unit area (veh/mi$^2$)</td>
</tr>
<tr>
<td>$k_j$</td>
<td>Jam density per unit area (veh/mi$^2$)</td>
</tr>
<tr>
<td>$\Omega$</td>
<td>Jam density in the absence of curbside parking (veh/mi$^2$)</td>
</tr>
<tr>
<td>$P_{\text{max}}$</td>
<td>Maximum number of parking spaces that could be accommodated by the street per unit area (space/mi$^2$)</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>Equivalency factor for converting the stock of cruising cars $C$ to an equivalent in-transit passenger cars $T_p$</td>
</tr>
<tr>
<td>$\beta$</td>
<td>Equivalency factor for converting the stock of commercial vehicles $T_c$ to an equivalent in-transit</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>Equivalency factor for converting the stock of double-parked vehicles $H$ to an equivalent stock of in-transit</td>
</tr>
</tbody>
</table>
The travel speed \( v \) could be expressed as \( v = v_0 \left( 1 - \frac{k}{k_j} \right) \), where \( v_0 \) is the free flow travel speed and \( K \) is the traffic density per unit area and \( K_j \) is the jam density. Accordingly, the travel time per unit distance \( t \) (which is the reciprocal of the travel speed \( v = 1/t \)) could be expressed as:

\[
t = \frac{t_0}{1 - \frac{k}{k_j}}
\]  

(1)

The density of cars per unit area \( k \) could be expressed as the sum of densities of the four types of travelers occupying the street space \( T_p, C, T_c, \) and \( H \). However, before we sum them we need to convert these different types of densities to an equivalent in-transit passenger car density \( T_p \) using equivalency factors \( \alpha, \beta, \) and \( \gamma \). The first factor \( \alpha \) is used to account for the effect of cruising vehicles. Drivers searching for parking travel at slower speeds relative to other road users while they hunt for spaces closer to their destination, \( \alpha \) is used to account for this effect it is to have a value of 1.5 similar to AI06, which indicates that a cruising car contributes 1.5 times as much as a car in-transit. The second factor \( \beta \) is used to account for the effect of the stock of in-transit commercial vehicles. There are ample references to estimate this value in accordance with prevailing site conditions in the study area such as the Highway Capacity Manual (HCM). We have assumed a default value of 1.8 for this factor. Finally, the third factor \( \gamma \) accounts for the effect of double-parking and we discuss it in the next section.

Therefore \( k \) can be expressed as:

\[
k = T_p + \alpha C + \beta T_c + \gamma H
\]  

(2)

The jam density \( k_j \) is affected by the proportion of the street area assigned to parking spaces. The more road space that is allocated to parking, the lesser is the available street area for traveling vehicles. This relation could be modified from AI06 as follows where \( P_{\text{max}} \) is the number of parking spaces that could hypothetically be available if all the street area was allocated to parking (with no area left for travelling cars).

\[
k_j = \Omega \left[ 1 - \frac{P_p + \theta P_c}{P_{\text{max}}} \right]
\]  

(3)

It can be seen from equations (1), (2), & (3) that the travel time per unit distance \( t(T_p, T_c, C, P_p, P_c, H) \) is an increasing function in the vehicles’ densities per unit area \( T_p, T_c, C, H \), and is also an increasing function in the stock of assigned parking spaces \( P_c, P_p \).

### 3.3 Modeling the effect of double-parking

In equation (2), we introduced gamma \( (\gamma) \) as a new factor to convert the stock of double-parked vehicles to an equivalent in-transit vehicles. To estimate the value of gamma, we contemplate the
effect of double-parking as a temporary lane drop that creates a bottleneck in traffic flow and we carry a standard bottleneck analysis such as explained in (May, 1990). The outcome of the analysis helps identify the traffic density in the congested area upstream of the bottleneck. Gamma ($\gamma$) is considered as the ratio between this density and the density of traffic in the uninfluenced area.

Consider for example a three-lane road section, where at one location a vehicle stops and double-parks occupying a travelling lane. This incident reduces the capacity of the road at this section to two lanes. Consider locations “A, B, C, D” shown in (Figure 4) location “A” at the upstream end and away from the influence of the bottleneck. Location “B” is also upstream but just before the bottleneck; location “C” is adjacent to the double-parked vehicle where the lane drop takes place.

And finally, location “D” is downstream of the double-parked vehicle where the road capacity returns to three-lane capacity. Consider the flow at location “A” is equivalent to 2.5 lanes capacity, which is lower than the capacity of the three-lane section, and assuming Greenshield’s relationship holds, the traffic state at location “A” must be on the low density leg of the flow-density curve as shown on (Figure 4). Now, consider the flow at location “C”, right at the bottleneck area, before we move back to location “B”. At location “C”, the flow-density curve is different from the three-lane section because it is only two lanes therefore the capacity and the jam density are two-thirds of their corresponding values on the bigger curve. The traffic flow at this section must drop from 2.5 lanes capacity to the maximum capacity of the two lanes section. Location “B” just upstream of the bottleneck is on the three-lane section. However, it is influenced by the bottleneck and therefore the flow on location “B” is equal to the flow on location “C”, and since this section represents a congested zone it necessarily falls on the right arm of the flow-density curve as shown in (Figure 4).

Finally, we consider gamma as the ratio between the traffic densities at location “B” which is influenced by the bottleneck due to the double-parking incident and location “A” at the uninfluenced upstream end:

$$\gamma = \frac{d_B}{d_A}$$

where:

$d_B$ = density at location B veh/mi

$d_A$ = density at location A veh/mi

---

For illustration, assume the following numbers are applicable for an urban three-lane road section in downtown:

Lane capacity \( q_m = 660 \text{ vph/lane} \) (or 1980 vph for 3 lanes)
Free flow speed \( u_f = 20 \text{ mph} \)
Jam density \( d_{jam} = 176 \text{ veh/mi} \cdot \text{lane} \) (or 528 veh/mi for 3-lanes)

And consider the flow-density relationship in equation (5) based on Greenshield applies:

\[
q = u_f d - \left( \frac{u_f}{d_{jam}} \right) d^2
\]  

(5)

where:

\( d \) = traffic density
\( u_f \) = free flow speed
\( d_{jam} \) = jam density
If flow at location “A” equal 2.5 lanes capacity then \( q_A \) equal 1650 vph. Substituting \( q_A \) in equation (5) and solving for the density \( d \), we get two solutions for \( d_A \), 102.33 or 425.67 veh/mi. And since location “A” resembles uncongested traffic flow, therefore \( d_A \) equals 102.33 veh/mi.

At location “B” the flow must drop to two-lane capacity, therefore \( q_B \) must equal 1320 vph/mi, substituting in (5) we get the density on the congested arm of the curve equal to 450.67 vph/mi. Gamma therefore is estimated as \( \gamma = \frac{450.67}{102.33} = 4.4 \).

### 3.4 Analysis of equilibrium

The model considers saturated parking in a steady-state traffic flow. Saturated parking indicates a demand that is high enough such that parking spaces remain 100% occupied during the study period, so as soon as one spot is vacated it is taken by another cruising car. A steady state flow, on the other hand, assumes a stationary environment where the traffic inflow into the system equals the traffic outflow. The steady state saturated parking equilibrium is demonstrated in (Figure 5) and it can be described by four equilibrium conditions, a pair for each type of vehicle:

For passenger cars

\[
D_p = \frac{T_p}{m_p t(T_p, T_c, C, P_p, P_c, H)} \tag{6}
\]

\[
\frac{T_p}{m_p t(T_p, T_c, C, P_p, P_c, H)} = \frac{P_p}{l_p} \tag{7}
\]

For commercial vehicles

\[
D_c = \frac{T_c}{m_c t(T_p, T_c, C, P_p, P_c, H)} \tag{8}
\]

\[
\frac{T_c}{m_c t(T_p, T_c, C, P_p, P_c, H)} = \frac{P_c}{l_c} + \frac{H}{l_c} \tag{9}
\]

Equation (6) for passenger cars and equation (8) for commercial vehicles define \( D \) the flow of vehicles entering the in-transit pool per unit area as equal to the flow of vehicles exiting the in-transit pool per unit area \( T/m_t \). Recall that \( t \) is the travel time per unit distance hence we multiply it by the travel distance \( m \) to get the total time spent in-transit (\( m_t \)).
Equations (7) & (9) describe the saturated parking equilibrium. For passenger cars, equation (7) states that the exit rate from the in-transit pool $T_p/m_p t$ (as discussed above) is now considered to be the entry rate into the cruising for parking pool. And as mentioned earlier, cars will continue to cruise until a space is open, so the exit rate from cruising from parking could be defined in terms of parking spaces per unit area and parking duration as $P_p/l_p$ (AI06). As shown in (Figure 5) $P_p/l_p$ in this case also defines the entry and exit rates from the parking pool.

Equation (9) defines the parking equilibrium condition for commercial vehicles. It terms the double-parking behaviour of commercial vehicles. The vehicles exiting the in-transit pool are ones that have arrived to destination and would need to park, the exit rate from the in-transit pool is $T_c/m_c t$. If parking spaces are available in close vicinity to destination then they park at the available space. However, if parking spaces are not available then they double-park near the destination but would not cruise for an empty space. $P_c$ is the stock of parking spaces assigned to commercial vehicles per unit area and $H$ is the stock of double-parking commercial vehicles per unit area. Accordingly, the entry rate into the parking pool is $P_c/l_c$ where $l_c$ is the average parking duration of commercial vehicles, and the remaining stock comprise the entry rate into double parking is $H/l_c$.

**Figure 5**
Saturated Parking in a Steady State Flow

**Passenger Cars**

- Demand inflow $D_p$
- Stock of in-transit PCs $T_p$
- Exit rate from in-transit $= T_p/m_p t$
- Stock of cruising PCs $C$
- Exit rate from cruising $= P_p/l_p$
- Stock of parked PCs $P_p$

**Commercial Vehicles**

- Demand inflow $D_c$
- Stock of in-transit CVs $T_c$
- Exit rate from in-transit $= T_c/m_d t$
- Stock of parked CVs $P_c$
- Entry rate into parking $= P_c/l_c$
- Stock of double-parked CVs $H$
- Entry rate into double-parking $= H/l_c$
- Exit rate $= H/l_c$

---

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3.5 Trip demand in the model

In the model, passenger car demand per unit area $D_p$ is assumed to be elastic (price sensitive); which is more true-to-life when compared to a fixed demand assumption. We used the same formula presented in Al06 except for the assumptions related to the travel time $t$, which now incorporates the effect of commercial vehicles as well as passenger cars as explained in section (3.2). Here we describe this demand function and how it connects other variables used in the model.

A Cobb-Douglas formula of the form $D_p = D_o F^e$ was used to define the relation between the trip demand and the trip price, where $F$ is the full trip price and $e$ is the demand elasticity with respect to price.

$F$ comprises three cost components: the in-transit travel time $m_p t$, the cruising for parking travel time $C l_p / P_p$, and the parking fee $f l_p$. The first and last terms are understandable; the cruising for parking time could be explained as follows:

$P_p / l_p$ is the rate of vacating a parking spot per unit area. Since generally time is the reciprocal of rate, therefore when $C$ is multiplied by the reciprocal of the vacating rate, which is $l_p / P_p$, we get the expected cruising time that each driver faces.

The value of time $\rho_p$ is used to convert the time components to equivalent dollar cost. The demand function could therefore be written as:

$$D_p = D_o \left[ \rho_p m_p t + \rho_p C \left( \frac{l_p}{P_p} \right) + f l_p \right]^e$$

(10)

where:

- $D_p =$ passenger car trip demand per unit time-area (veh/hr-mi²)
- $D_o =$ constant calibrated depending based on actual demand in study area
- $m_p =$ distance travelled by passenger cars in the downtown area to destination (mi)
- $C =$ stock of cruising passenger cars per unit area (veh/mi²)
- $P_p =$ parking spaces allocated to passenger cars per unit area = stock of cars parked (veh/mi²)
- $l_p =$ parking duration of passenger cars (hr)
- $f =$ on-street parking fee per unit time ($/hr)
- $e =$ Elasticity of demand with respect to trip price

It is important to mention here that while we have assumed an elastic demand function for passenger cars, the same is not valid for commercial vehicles. It is far less sensitive to the price attached to on-street parking and even to the availability of such parking. As explained earlier, due to
the high value of time attached to commercial vehicles’ trip cost, if curbside parking is not available then double-parking would be the cheapest available alternative. A recent study by Tipagornwong and Figliozzi (2015)8 suggests that in many cases double-parking fines are already built-in to the cost of delivery.

3.6 Analysis of social optimum

In this section we take a policy maker’s perspective and examine the social costs and social benefits entailed when on-street parking is contemplated.

First we consider passenger cars. The average cost of a passenger car trip is the sum of four components: the cost of in-transit travel time, the cruising for parking time, the cost of parking fee, and finally, since we are considering social costs, the opportunity cost of time at destination.

\[
\text{Average cost of passenger car trip} = \rho_p m_p t + \rho_p C \left( \frac{l_p}{P_p} \right) + f l_p + \rho_p l_p
\]  

(11)

The total cost is calculated by aggregating the average cost of all passenger car trips. This is achieved by multiplying the average cost by the flow per unit area. Since parking is saturated and since a steady-state traffic flow is assumed, the flow per unit area must be equal to \( P_p / l_p \). This was illustrated in (Figure 5) where the demand inflow \( D_p \) is equal to the exit rate \( P_p / l_p \).

The total cost, or social cost, of passenger cars can therefore be written as:

\[
\text{Total cost of passenger car trip} = \rho_p T_p + \rho_p C + f P_p + \rho_p P_p
\]  

(12)

Likewise, the average cost of commercial vehicles is broken-down to four components. The cost of in-transit travel time, the cost of parking fee applied only to the proportion of vehicles that park \( \frac{P_c}{P_c + H} \), the cost of double-parking fine which applies only to the proportion of vehicles that double-park \( \frac{H}{P_c + H} \), and finally the opportunity cost of time at destination.

\[
\text{Average cost of commercial vehicle trip} = \rho_c m_c t + f l_c \left( \frac{P_c}{P_c + H} \right) + q l_c \left( \frac{H}{P_c + H} \right) + P_c l_c
\]  

(13)

where:

- \( q = \text{double-parking fine per unit time} \)

In a steady-state environment the flow of commercial vehicles per unit area \( D_c \) is equal to the combined exit rates of parked vehicles \( P_c / l_c \) and double-parked vehicles \( H / l_c \). This is also illustrated in (Figure 5). Taking the flow as \( \left( \frac{P_c + H}{l_c} \right) \) and multiplying it by the average cost in (13), the total cost of commercial vehicles trips can then be written as:
Total cost of commercial vehicles trip = \( \rho_c T_c + fP_c + qH + \rho_c(H + P_c) \)  \( (14) \)

Finally, the total social cost of both passenger cars and commercial vehicles is the sum of (12) and (14):

Total cost = \( \rho_p T_p + \rho_p C + fP_p + \rho_p P_p + \rho_c T_c + fP_c + qH + \rho_c(H + P_c) \)  \( (15) \)

The total social benefit \( B \) is likewise the sum of passenger cars and commercial vehicles benefits 

\[ B_p = \int D_p^{-1} \text{ and } B_c = \int D_c^{-1}. \]

The social surplus equals the social benefit minus the social cost:

\[ SS = \int D_p^{-1} + \int D_c^{-1} - \left[ \rho_p T_p + \rho_p C + fP_p + \rho_p P_p + \rho_c T_c + fP_c + qH + \rho_c(H + P_c) \right] \]

\( (16) \)

Therefore, the social optimum is the set \((D_p, T_p, C, P_p, T_c, H, P_c, f)\) that maximizes the social surplus subject to the equilibrium conditions. Noticing that \( D_c \) is fixed, the maximization objective could be written as:

\[ \max_{D_p, T_p, C, P_p, T_c, H, P_c, f} \int D_p^{-1} + \int D_c^{-1} - \left[ \rho_p T_p + \rho_p C + fP_p + \rho_p P_p + \rho_c T_c + fP_c + qH + \rho_c(H + P_c) \right] \]

\( s.t. \)

\[ D_p = D_o \ast \left[ \rho_p m_p t + \rho_p C \left( \frac{l_p}{P_p} \right) + f l_p \right]^e \]

\[ D_p = \frac{T_p}{m_p t(T_p, T_c, C, P_p, P_c, H)} \]

\[ D_c = \frac{T_c}{m_c t(T_p, T_c, C, P_p, P_c, H)} \]

\[ t = \frac{t_o}{1 - \frac{k}{k_j}} \]

An examination of the objective presented in (17) indicates that the optimization process should tend to clear the stock of double-parking vehicles followed by the cruising vehicles as they produce
the highest costs. One way they incur higher costs compared to in-transit vehicles is through their effect on the traffic density \( k \) as defined in equation (2), which adversely affects the travel time \( t \). In the case study in section 5 we demonstrate this optimization and compare it to the equilibrium that takes place before optimization.

In the next section we demonstrate how the proposed model compares to the base model of (Arnott and Inci 2006).
4. Model Verification

The previous section explained the components of the model; here we attempt to show how the proposed model establishes a generalization of Arnott and Inci model. In the base case where no commercial vehicles are considered, both models produce the same results, whereas when commercial vehicles are considered the proposed model is capable of incorporating their effect and providing a fuller view of the actual road users and their implication on road congestion.

4.1 Discussing policy inputs

We compare the application of both models through studying the state of equilibrium that takes place when given a specific parking policy. This is especially useful when examining current policies’ effects and comparing it to proposed improvements.

To analyze a specific policy using the model, the required inputs include the parking fee $f$, the allocated parking spaces per unit area ($P_p$ and $P_c$), and the measured commercial vehicle demand per unit area $D_p$. The resulting traffic state could be defined in terms of the travel time $t$, the traffic densities $T_p$, $T_c$, $C$, $H$, and the expected passenger car traffic flow $D_p$ (all normalized per unit area). These six variables are the postulated unknowns in the system of equations (1, 6, 7, 8, 9, &10) explained earlier, which mainly comprise the conditions of equilibrium and the travel time and demand functions.

For comparison purpose we use the same calibration values used in Arnott and Inci model. It was based on a study-area featuring 64 blocks per square mile, and an assumed 58 parking spaces per block, so the total available parking spaces per square mile is 3712 spaces. If hypothetically all the street area was assigned to parking (with no street area left to travelling cars) then this would yield a max number of parking spaces $P_{max} = 11,136$. Therefore the ratio of the allocated parking area to the total street area is $P/P_{max}$ is $3712 / 11136 = 0.33$. This ratio is used in equation (3) to estimate $k_j$.

Other parameters used in the model include the demand function constant, $D_0$, which was chosen as 3190.04 assuming a base trip price of $F = 15$ and $\Omega$ as 2667.2, assuming that 30 percent of cars are cruising for parking. The free flow travel speed is assumed to be 20 $mph$, which corresponds to a free flow travel time per unit distance of 0.05 $hr/mi$. And finally, the elasticity of trip demand with respect to trip price is assumed to be 20 percent, $e = -0.2$. In all scenarios we hold the parking fee fixed at $f = $1/hr to enable comparison of these cases. We discuss fee optimization in a later section.

4.2 Results

(Figure 6) displays the outcome of Arnott and Inci model in a base scenario and the outcome of the proposed model in the same scenario in addition to two other scenarios.
The base scenario represents a case where no flow of commercial vehicles are allowed i.e. \( D_c = 0 \). In such case the outcomes of both models are identical since the effects of commercial vehicles are non-existent, and this could be seen by comparing the first two columns (Figure 6).

In scenario 1, commercial vehicles were introduced with \( D_c = 250 \text{ veh/hr/mi}^2 \) and parking period \( l_c = 0.15 \text{ hr (9 min)} \) and an in-transit travel distance between stops \( m_c = 0.181 \text{ mile} \). Yet with the introduction of the commercial vehicles in this scenario no parking was assigned to it. The proposed model now shows the corresponding density of commercial vehicles in-transit per unit area \( T_c = 13.34 \) and the density of double-parked vehicles per unit area \( H = 37.5 \text{ veh/mi}^2 \).

The model also evaluates the new densities of in-transit and cruising passenger cars \( T_p \) and \( C \) that takes place with the introduction of commercial vehicles and their parking behaviour. The new \( T_p \) has increased while \( C \) has reduced; this is a direct reflection of the increased trip price. The increase came mainly in terms of an increased travel time \( t(T_p, T_c, C, P_p, P_c, H) \).

In the last scenario, we maintain the introduced flow of commercial vehicles but now we assign part of the parking spaces to commercial vehicles. The corresponding equilibrium shown in the last column of the table shows a reduced travel time \( t \) as well as reduced double-parking vehicles per square mile \( H \). The above examination shows how the proposed model provides a new set of tools for policy makers to evaluate the actual effect of parking polices on road users and traffic congestion.
It is noted, however, that the choice of parking fees and parking space allocation examined in the previous table does not appear to be clearing the double-parking vehicles nor does it appear to be clearing the cruising for parking in the case of passenger cars. In section five we demonstrate in a case study how the model is applied to optimize the social surplus.
5. Case Study of Toronto Parking Pricing and Allocation

Toronto is Canada’s largest economic center and most populous city. Haider et al. (2009)\(^9\) carried an analysis for a segment in downtown Toronto and found that around 80,000 packages and parcels are delivered to that part of the downtown in a given day. The study points to the inadequate supply of infrastructure necessary for the freight industry to deliver packages and parcels to consignees in an efficient manner without disrupting the traffic.

In this section we consider part of downtown Toronto shown in (Figure 7) to demonstrate the application of the model and how useful it could be in creating significant gains in social surplus.

5.1 Field data

The chosen area comprises the Financial District in downtown Toronto; it is bound by Simcoe St. and Victoria St. from east and west, and Queen St. and Front St. from north and south. The area is known to be the most densely built-up area of Toronto and is home to numerous financial institutions, corporate headquarters, and key legal and accounting and insurance firms. It is also home to major hotels and retail stores. Among the towers and establishments found in this area are the Toronto Stock Exchange, one of the largest stock exchanges in the world, Toronto Board of Trade, Royal Bank of Canada, Toronto Dominion Bank, Bank of Montreal, Canadian Imperial Bank of Commerce, Scotia Bank, Trump International Hotel and Tower, and the Ritz Carlton. The Financial District attracts huge amount of commuter trips daily. According to Transportation Tomorrow Survey (TTS, 2015)\(^{31}\), traffic zones 54, 55, 56, & 57 in the city of Toronto, which include the study area, receives 131,801 trips every day (including trips made by transit) which puts this area among the top in the whole of Toronto.

---

The study area has an area of almost 0.2923 square miles and within the perimeter of it, there is a total lane length of 19.836 lane miles and if we remove the parts occupied by the intersections from this number it becomes 18.845 lane miles.

To find the total street area we multiply this length by the typical lane width in the downtown, which is 0.00211 miles (3.4 m), the total street area is therefore almost 0.0398 square miles (102,890 square meters).

According to the city by-law number 569-2013, the minimum length of a passenger car parking space considering parallel parking is 6.7 m. And we assume that the width of a parking space is that of a typical street lane in Toronto downtown which is 3.4 m.

With the above information we could find $P_{\text{max}}$ by dividing the total street area by the area of a single parking space, which generates almost 4,517 parking spaces in the study area and when normalized per square mile we get $P_{\text{max}} = 15,452 \text{ space/mi}^2$.

We assume that one-fourth of the street area is allocated to on-street parking so $P = \frac{15452}{4} = 3863 \text{ spaces/mi}^2$.

$\Omega$ is calibrated assuming that at jam density the headway distance between vehicles is 30 ft and therefore the jam density per lane per mile stands at 176 vehicles, which is within standard range as mentioned for example in (May, 1990). $\Omega$ could then be estimated as the product of the total lane...
miles 18.845 times 176 and then divided by the area of the study area (0.2923) gives \( \Omega = 11,346.97 \).

The demand function constant \( D_0 \) is calibrated at 3319.8 for the study area assuming a base trip price \( F = 15 \). For the on-street parking fee in this area we considered the current rates, which is metered using pay-and-display devices, and it charges $4/hr.

For data related to the commercial vehicles, we referred to the Cordon Count Data Retrieval System (CCDRS) managed by municipalities that include the City of Toronto, and the Regional Municipalities of Durham, Halton, Peel and York and the Ministry of Transportation Ontario, to obtain an estimated truck flow in the study area of \( D_c = 865 \text{ veh/hr} \cdot \text{mi}^2 \).

The value of time of commercial vehicles is considered to be $110/hr based on a study by Ismail, Sayed, and Lim (2009)\(^{32}\). Double-parking fine is assumed to be \( q = $150 \), which is the minimum fixed parking fine at busy streets in Toronto.

The average parking duration for commercial vehicles is assumed to be 9min (0.15 hr). Finally, we consider the commercial vehicle parking space dimension to follow the requirements of Type B of the city by-law number 569-2013, which is 11m in length. Using this information we can estimate \( \theta \), the ratio between the parking space dimensions of commercial vehicle and that of passenger cars as \( \frac{11}{6.7} = 1.64 \).

\(^{32}\) Ismail, K., Sayed, T., Lim, C., 2009. A study of the commercial vehicle value of time for operation at border crossings. In the Annual Conference of the Transportation Association of Canada.
5.2 Results and discussion

Figure 8
Optimization Results – Social Optimum vs. Equilibrium – Case study

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Solution

| $P_p^*$ (space/mi$^2$)                     | -         | 3650           | 4406           |
| $P_c^*$ (space/mi$^2$)                    | -         | 130            | 130            |
| $f^*$ ($/hr$)                              | -         | 8.93           | 2.86           |
| $D_p$ (veh/hr/mi$^2$)                      | 1932      | 1825           | 2203           |
| $T_p$ (veh/mi$^2$)                        | 233.99    | 186.93         | 227.19         |
| $C$ (veh/mi$^2$)                          | 442.02    | 0              | 0              |
| $T_c$ (veh/mi$^2$)                        | 9.48      | 8.02           | 8.07           |
| $H$ (veh/mi$^2$)                          | 129.75    | 0              | 0              |
| $t$ (hr/mi)                               | 0.0606    | 0.0512         | 0.0516         |
| $v$ (mi/hr)                               | 16.5      | 19.5           | 19.4           |

Gain in social surplus $\Delta SS$ ($/hr-mi^2$)

|                | $13,502$ | $23,204$ |
(Figure 8) compares the outcome of three cases applied to the study area. In the first case, we consider the base case equilibrium that takes place with current parking rates and the allocation of all parking spaces to passenger cars only. In the second scenario we apply the model to optimize the social surplus while holding the parking spaces fixed. The outcome of such optimization shows how best to allocate the available parking spaces between commercial and passenger cars and the corresponding optimum parking fees that clears cruising for parking.

In the first case, the base case equilibrium results show that despite a realized passenger demand of $1932 \text{veh/}hr/\text{mi}^2$ compared to truck delivery demand of $865 \text{veh/}hr/\text{mi}^2$, the resulting stock of road traffic is $T_p = 233.99 \text{veh/}mi^2$ compared with $T_c = 9.48 \text{veh/}mi^2$. This result is interesting because a typical road traffic count would indeed show about 5% truck traffic typically, and in this case we obtain 3.9%. However, it turns out that much of the truck demand is being allocated to double-parking for deliveries.

The implication of ignoring commercial vehicle on-street parking is evident on the relatively high density of double-parking vehicles on the streets with $H = 129.75 \text{veh/}mi^2$. On the other hand, the applied parking fee did not eliminate all the cruising for parking as $C = 442.02 \text{veh/}mi^2$, which is about 54% of the total road users in the road space allocated to travelling. The higher cruising stock in this case is a result of ignoring the traffic entering the downtown looking to park in garage parking. Incorporating this traffic would reduce the relative proportion of cruising vehicles. For example, if street parking is comprised of only half the traffic coming to downtown with the other half finding garage parking, then the actual cruising proportion should be closer to 27%. Since we don’t have data on that information, we cannot properly assess the cruising proportion. The congestion in this scenario is evident in the lower travel speed, which stands at 16.5 mi/hr.

In the second scenario, the allocation of the fixed parking spaces is optimized, resulting in 130 spaces/\text{mi}^2 assigned to commercial vehicles and 3650 spaces/\text{mi}^2 to passenger cars. Parking fees are also optimized at $8.9/\text{hr}$. With this policy the travel speeds rise to an average of 19.5 mi/hr and the total gain in social surplus compared to the initial equilibrium is $13,502$ per hour per square mile. Both the cruising and double-parking are eliminated.

The last case demonstrates the social optimum under first-best allocation, where parking spaces are not fixed anymore. Applying the proposed model, the optimum passenger car parking spaces is $P_p = 4406 \text{spaces/}mi^2$ and for commercial vehicles $P_c = 130 \text{spaces/}mi^2$. By allowing total parking spaces to increase by 20%, the parking fee can be reduced down to $f = 2.86/\text{hr}$. This trade-off between parking fee and space availability fits with Arnott and Inci (2006). The total gain in social surplus from this policy compared to the initial case is $23,204$ per square mile per hour.

The relative proportion of each segment of road users with respect to total road users is best demonstrated in Figure 9. The first stacked bar represents the first scenario, while the other two bars represent the optimized scenarios. The height of each bar reflects the total number of vehicles that result from each policy. The first bar shows the resulting densities of four segments of road users,
these are the stocks of in-transit passenger and commercial vehicles $T_p \& T_c$ as well as the stocks of cruising and double-parking vehicles $C \& H$. The congestion in this scenario is evident in the total height of the bar which indicates the highest aggregate stock of vehicles on the street among the three scenarios. The second and third bars indicate the resulting lower densities of vehicles on the street as a result of the optimized policies, it is also noted that cruising and double-parking vehicles are cleared in the latter scenarios.

**Figure 9: Mixture of Road Users Across the Three Scenarios**

The above results demonstrate that the current practice of disregarding the effect of commercial vehicles and their parking behaviour on congested downtown street networks has inevitably lead to devising inefficient solutions to meet the congestion. For policy makers to be able to best respond to congestion problems it is necessary to capture the effect of all road users including commercial vehicles. The case study demonstrates how developing an inclusive policy leads to considerable efficiency gains, which is much needed on the streets of the busy downtown centers.
6. Conclusion

It is well established that urban truck deliveries make a big impact on commuter parking, because of the shared use of parking spaces, the inelasticity of freight demand, and the need to double-park when no spaces are available due to need for proximity. Nonetheless, the literature on downtown on-street parking generally continues to exclude truck delivery behaviour. The few studies of truck deliveries are simulation based or do not integrate with commuter parking.

In this study, we present an analytical equilibrium model that evaluates the effects of different parking policies in urban centers with respect to network congestion, cruising, double-parking, and the travel behaviour of commercial and passenger vehicles. It is the first such model, and also the first analytical evaluation of downtown Toronto parking pricing and space allocation policies. The parking model is shown to be a generalization of the commuter equilibrium model from Arnott and Inci (2006), one that can also capture a truck delivery fleet class that is inelastic to traffic conditions and double-parks when no spaces are available.

The case study makes several key findings. First, we measured and estimated parameters of the model for the Financial District in downtown Toronto such that a baseline scenario is defined. This baseline scenario can serve as a benchmark for policymakers to consider different policies. From the baseline, we considered two policy tools. The first is to price and allocate the existing parking spaces to trucks to optimize social surplus (maximize the total benefit minus the total cost). We find that increasing the parking fee from $4/hr to nearly $9/hr and assigning 3.4% of parking spaces to truck parking would eliminate cruising and truck double-parking, resulting in a social surplus gain of over $13,500/hr/mi².

Under a first-best allocation policy where the total number of parking spaces can also change, we find that it is optimal to increase number of parking spaces by 20% (of which truck parking spaces would constitute 2.9% of spaces), and reduce parking fees to under $3/hr, we can eliminate cruising and truck double-parking while increasing social surplus to $23,200/hr/mi².

The model helps policy makers develop strategies to improve urban parking policies by being able to plan and optimize trade-offs in parking spaces, prices, and network congestion.

Commercial vehicles serve financial and commercial institutions in the downtown and it constitute a segment of road users that is frequently ignored by both policy makers and researchers. However, efficient solutions to congestion problems must capture all segments of road users to be able to respond with proper polices. The continued double-parking behaviour of commercial vehicles shows that ignoring this segment and resorting to traffic fines might not provide the sought efficiency in the network. It is therefore necessary to incorporate this segment with other road users and devise inclusive policies. In the case study we have demonstrated how the developed model captures all the segments of roads users and optimizes the road space accordingly, allowing the most efficient allocation of on-street parking and the optimum corresponding parking fees.
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