By: Niruban Arulselvan

LP Sensitivity Analysis

Maximize $f = 2x + 9y$ given the following constraints

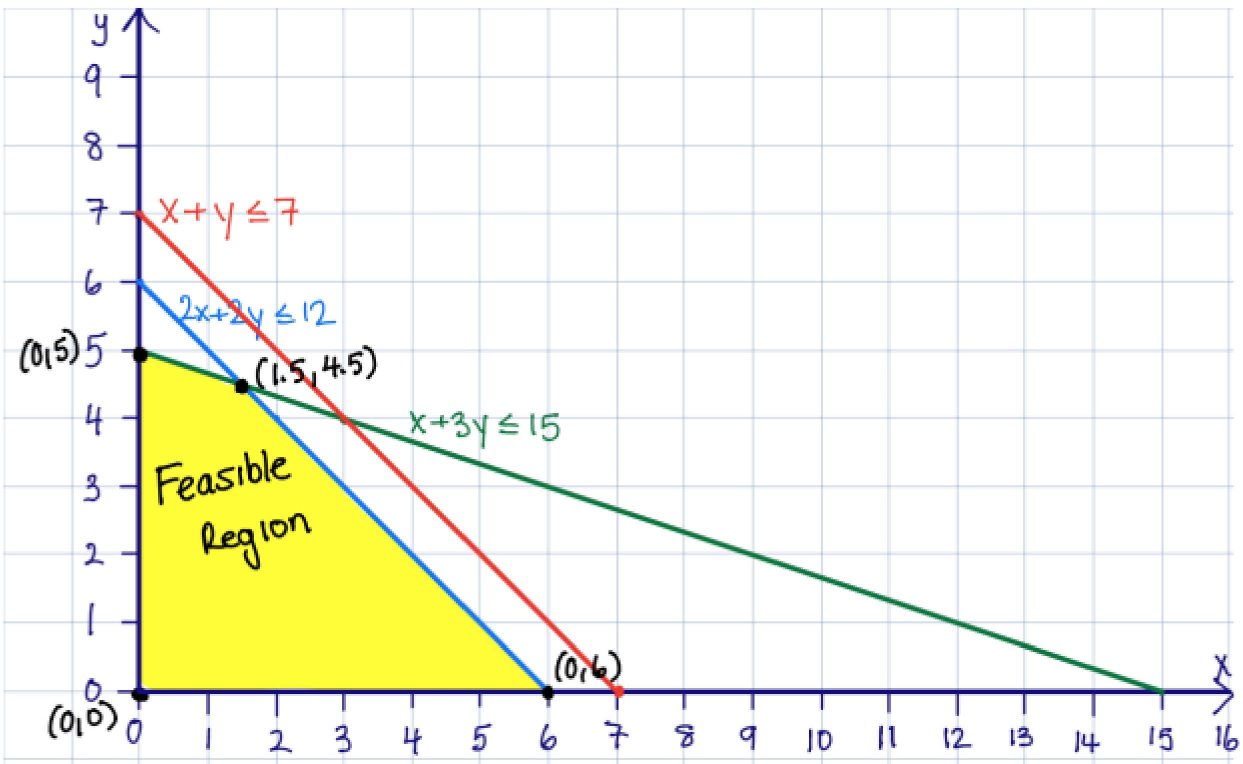
$$x + y \leq 7$$
$$2x + 2y \leq 12$$
$$x + 3y \leq 15$$

Introductory Steps

Step One: Graph the inequalities

Step Two: Determine the **feasible region**

- **Use (0,0) as the test coordinate**
- **Mark the coordinates**



Corners: (0,0), (0,5), (6,0), (1.5, 4.5)

Step 3: Find the Maximum and Optimal Point

- Substitute each corner point into the objective function. The highest value is the maximum value. Do this for every corner value.
  - (0,0) = 2(0) + 9(0) = 0
  - (0,5) = 2(0) + 9(5) = 45
  - (6,0) = 2(6) + 9(0) = 12
  - (1.5,4.5) = 2(1.5) + 9(4.5) = 43.5
- Maximum Value = 45, Optimal Point = (0,5)

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Identify Binding and Non-Binding Constraints

Step One: Substitute the optimal point into all the constraints
Optimal Point is (0,5)

$x + y \leq 7 \rightarrow 5 \leq 7$ (Non-Binding Constraint)
$2x + 2y \leq 12 \rightarrow 10 \leq 12$ (Non-Binding Constraint)
$x + 3y \leq 15 \rightarrow 15 \leq 15$ (Binding Constraint)

Step Two:

- If your answer is equal to the Right Hand Side (RHS) of the inequality, then the constraint is **BINDING**.
- If your answer is not equal to the RHS of the inequality, then the constraint is **NON-BINDING**.
- If your answer is greater than the RHS of the inequality, you've made an error with the beginning steps, go back and correct the error.

Find Redundant Constraints

- Any lines that are NOT on the boundary of the feasible region as shown in the graph above, are REDUNDANT CONSTRAINTS
- $x + y \leq 7$ IS A REDUNDANT CONSTRAINT

Find the Range of Optimality of the Objective Coefficient

Step One: Find the slope of the objective function.

Objective Function: $f = 2x + 9y$

- Set the objective function into the notation of Y = Mx + B, where m represents slope.

M_f (slope of objective function)= $\frac{-2}{9}$

Step Two: Look at the graph and find the slopes of the lines that INTERSECT with the optimal point (0,5).

The range of optimality is expressed as $\boldsymbol{m_1 \leq m_f \leq m_2}$
$\boldsymbol{m_1}$= Smaller Slope
$\boldsymbol{m_f}$= Objective Function Slope
$\boldsymbol{m_2}$= Larger Slope

- Once you have the slopes of your OBJECTIVE FUNCTION and the slopes of the lines that intersect with your optimal point, you can find the RANGE OF OPTIMALITY.
- The RANGE OF OPTIMALITY is expressed in terms of x and y. This means that there are two answers.

There is only one line that intersects with the optimal point and falls in the feasible region, and that is $x + 3y \leq 15$

The slope of $x + 3y \leq 15$ is $\boldsymbol{m_1 = -\frac{1}{3}}$

Solve for X

Set the numerator of the objective function slope as x

$$-\frac{1}{3} \leq -\frac{x}{9}$$

$$\boldsymbol{x \leq 3}$$

Solve for Y

Set the denominator of the objective function slope as y

$$-\frac{1}{3} \leq -\frac{2}{y}$$

$$\boldsymbol{y \leq 6}$$

Therefore the RANGE OF OPTIMALITY IS $\boldsymbol{x \leq 3}$ AND $\boldsymbol{y \leq 6}$.

Find the Shadow Price

- What will happen to the objective function if the right side of the constraints increase by 1?
- You must find the shadow prices for BINDING CONSTRAINTS.
- NON-BINDING CONSTRAINTS always have a shadow price of 0.

Step One: Remember the binding and non-binding constraints that were found earlier in the problem. You can state that the shadow price of a non-binding constraint is 0 (there is nothing to solve).

$x + y \leq 7 \rightarrow 5 \leq 7$ (Non-Binding Constraint) Shadow Price = 0
$2x + 2y \leq 12 \rightarrow 10 \leq 12$ (Non-Binding Constraint) Shadow Price = 0
$x + 3y \leq 15 \rightarrow 15 \leq 15$ (Binding Constraint)

Step Two: For the only binding constraint, increase the RHS by 1. (Note: If there were two binding constraints, you would increase the RHS of the first one, leave the other one unchanged, then solve for X and Y as in Step 3. Then for the second binding constraint, increase the RHS of it by 1, leave the first binding constraint unchanged and proceed by solving for X and Y as in Step 3).

$$x + 3y \leq 15$$

$$x + 3y = 16$$

$$y \leq \frac{16}{3}$$

$$x = 0$$

Step Three: Solve for x and y of the binding constraint. Using either substitution or elimination.

$$x = 0, y = \frac{16}{3}$$

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Step Four: Substitute the values of x and y into the objective function and solve. We'll call this F_N or F (New).

$$F = 2x + 9y$$
$$F_N = 2(0) + 9(\frac{16}{3})$$
$$F_N = 48$$

Step Five: Subtract the New value from the Optimal value.

$$SP_3 = F_N - F_{Optimal}$$
$$48 - 45 = 3$$

Therefore, the shadow price of $x + 3y \leq 15$ is 3.
Since for this problem there is only one binding constraint, you would just do this process for that one. If there were more than one binding constraints,

Find the Amount of Change in Objective Function if RHS of Constant Increases by 8%.

Step One: Multiply the RHS of the binding constraint by 0.08.

$$x + 3y \leq 15 \times 0.08$$
$$k = 1.2$$

Step Two: Multiply the shadow price of the binding constraint by your answer from step one. The Triangle known as delta Δ represents change in.

$$\Delta Objective\ Function = SP \times k$$
$$\Delta Objective\ Function = 3 \times 1.2$$
$$\Delta Objective\ Function = 3.6$$