Winter 2019 QMS202 Hypothesis Testing Review

Chapter 11 Fundamentals of Hypothesis Testing: 1-Sample Tests
What is the hypothesis testing?
Hypothesis testing begins by taking a statement or claim then assessing it to prove whether it should be rejected or not rejected.

Null Hypothesis:

- Commonly accepted result or what "should" happen.
- It is described with the symbol → H_0

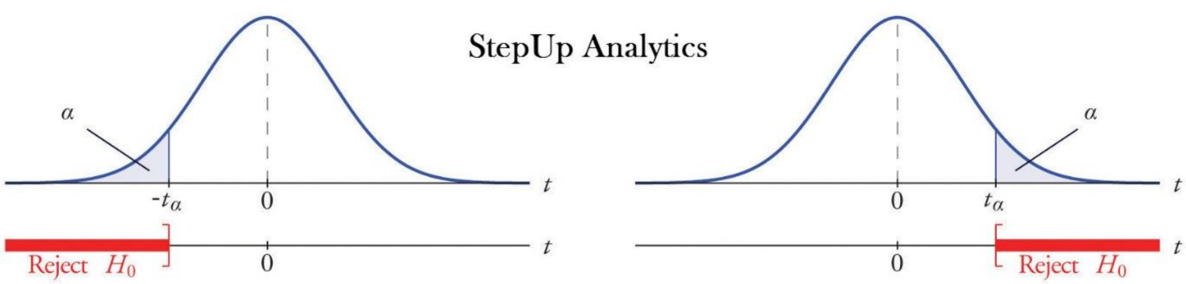
Alternative Hypothesis (<u>is what we input in the calculator</u>)

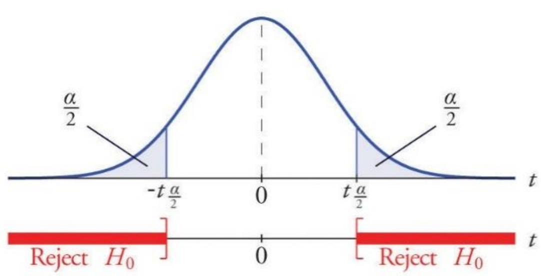
- Opposite of the null hypothesis
  - "If the null hypothesis is considered false, something else must be true"
  - The conclusion that is reached by rejecting the null hypothesis
- It is described with the symbol → H_a or H_1

HYPOTHESIS TEST:
Null (H_0) has equality → equal (=), at most (≤) or at least (≥)
Alternative: (H_a) has inequality → not equal (≠), less than (<), or greater than (>)

Types of Hypothesis Tests (using t Test as an example, same rules for Z Test)

Left (lower) -tail test: $H_\alpha : \mu < \mu_0$ Right (upper) -tail test: $H_\alpha : \mu > \mu_0$





Two-tail test: $H_\alpha : \mu \neq \mu_0$

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Hypothesis Testing One Sample Test Steps:

Step 1: Decide the claim H_0: (μ for the mean or π for the proportion) (=, ⩽, ⩾)

H_1 or Ha: (μ or π) (≠, <, >)

Step 2: Use given data to support your claim: σ, x̄, n
Step 3: Requirements:
i) level of significance is α ;
ii) type of test (2 tail test ≠, right tail >, left tail <)
iii) critical value z or t (to find critical values under DIST function on calculator)

Step 4a: Run the Calculator (for the One Sample Z Test)
1. TEST 2. "Z" → 1-S (for the mean) 1-P (for the proportion) 3. Variable
4. Choose a sign according to Ha: <, >, or ≠ 5. Input given data: σ, x̄, n
6. Press EXE → we get: Z = Z_{STAT} p = p-value
Step 4b: Run the Calculator (for the One Sample t Test)
1. TEST 2. "t" → 1-S (for mean)
3. Variable (when sx is given) List (when observations data is given)
4. Choose a sign according to Ha: <, >, or ≠ 5. Input given data: σ, x̄, n
6. Press EXE → we get: t = t_{STAT} p = p-value

P- Value VS. LEVEL OF SIGNIFICANCE α (Greek letter alpha)
P-value < α: Reject H_0 P-value ≥ α: Don't reject H_0

Conclusion
Reject H_0, since there's evidence to support Ha
Don't reject H_0: since there's no evidence to support the Ha

Errors in hypothesis testing:
Type I: Reject H_0, when H_0 is true → α
Type II: Don't reject or fail to reject H_0, when H_0 is false → β

	Actual Situation	
Statistical Decision	H_0 True	H_0 False
Do not reject H_0	Correct decision Confidence = (1 – α)	Type II error P (Type II error) = β
Reject H_0	Type I error P (Type I error) = α	Correct decision Power = (1 – β)

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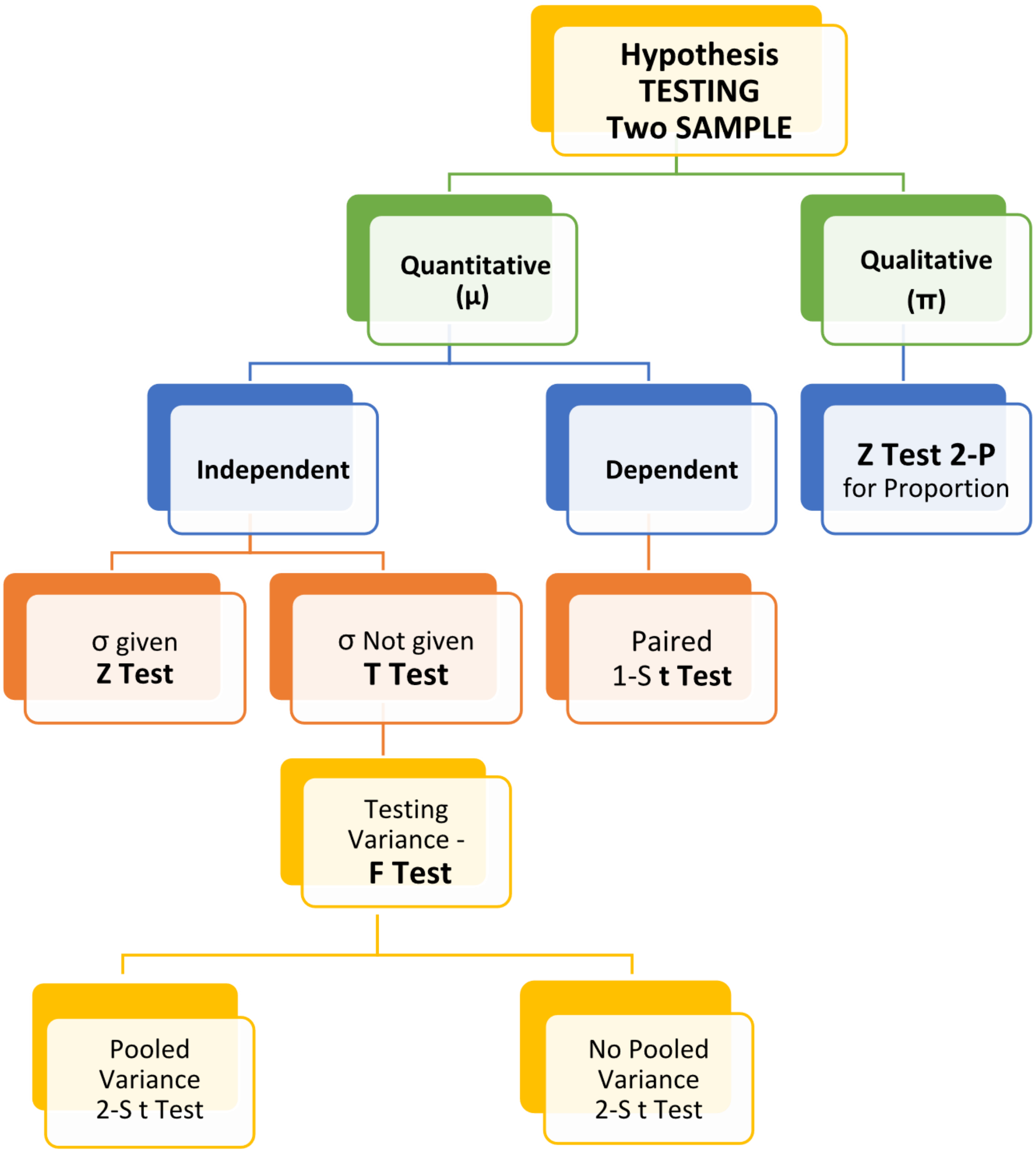
Chapter 12 Hypothesis Testing: Two-Sample Tests



The Hypothesis for Two-Sample Tests (using two-tailed test as an example)

Null Hypothesis H_0 : $\mu_1 = \mu_2$ or $\mu_1 - \mu_2 = 0$ (for 2 population means, Z Test or t Test);

$\Pi_1 = \Pi_2$ (for 2 population proportions, Z Test)

$\sigma_1^2 = \sigma_2^2$ (for 2 population variances, F Test)

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Alternative Hypothesis H_1 or H_a:

$\mu_1 \neq \mu_2$ or $\mu_1 - \mu_2 \neq 0$ (for 2 population means, Z Test or t Test);

$\Pi_1 \neq \Pi_2$ (for 2 population proportions, Z Test)

$\sigma_1^2 \neq \sigma_2^2$ (for 2 population variances, F Test)

After deciding the H_0 and H_1, we can use according TEST functions on the calculator to run the test and get the result numbers.

Calculator Tips :

Similar to the Step 4a and 4b on page 2, we now use 2-S for the mean under Z Test and t Test, 2-P for the proportion under Z Test, and F Test for the population variances.

For the 2-S t Test, if given the two samples have equal variance, press "Pooled: On",

If not given or given they have separate / unequal variances, press "Pooled: Off".

F Test for the Ratio of Two Variances (To determine if it is "Pooled On" or "Pooled Off" for the 2-S t Test)

Equation: $F_{STAT} = S_1^2 / S_2^2$, the F_{STAT} test statistic follows an F distribution (always right skewed and bulk on the left) with numerator df $n_1 - 1$ and denominator df $n_2 - 1$

F critical value:

1). For a two-tail test, the two critical values are F Upper ($F_U = F_{\alpha/2, n1-1, n2-1}$) and F Lower ($F_L = F_{1-\alpha/2, n1-1, n2-1} = 1/F_U$) Reject H_0 if $F_{STAT} > F_U$ or $F_{STAT} < F_L$, otherwise, do not reject H_0

2). For a lower/ left tail test, there is one critical value, $F_L = F_{1-\alpha, n1-1, n2-1}$

Reject H_0 if $F_{STAT} < F_L$, otherwise, do not reject H_0

3). For an upper/right tail test, there is one critical value, $F_U = F_{\alpha, n1-1, n2-1}$

Reject H_0 if $F_{STAT} > F_U$, otherwise, do not reject H_0

Tip: For F Test, input data in the testing list or the sample standard deviation S_1 (for sx1) and S_2 (for sx2), which are the square root of the sample variances S_1^2 and S_2^2

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Tip! For the Two dependent / paired Sample Test, we still need to use 1-S under t Test with the computed differences data with degrees of freedom (df) = n1 + n2 -2

When are the Two samples dependent?

Two Dependent / Paired Samples:
Type 1 – Repeated Measurement taken from same item/individual: Dependent samples are characterized by a measurement, then some type of intervention, followed by another measurement. (Before and after). E.g. A study wants to test how well runners perform with and without the new tech shoes. (still the same runners)

Type 2 – Matched samples according to some characteristics: because the same individual or item is a member of both samples. E.g. Consumers wish to know the differences in price of same items sold at two grocery stores. (still the same items)

Two Independent Samples:

The samples chosen at random are not related to each other. E.g. Study the average salaries of company X and firm Y and since a person cannot be an employee in both companies in most cases. (not the same employees)

Confidence Interval Estimate for the Mean Difference

$$\bar{D} - t_{\frac{\alpha}{2}} \times \frac{S_D}{\sqrt{n}} \leq \mu_D \leq \bar{D} + t_{\frac{\alpha}{2}} \times \frac{S_D}{\sqrt{n}}$$

Testing the Proportions of Two Independent Populations (using 2-P under Z Test)

If follows the same decision rules for 1 Sample Z Test mentioned before with

$$\bar{p} = \frac{X1+X2}{n1+n2} \text{ , } p_1 = \frac{X1}{n1} \text{ , } p_2 = \frac{X2}{n2}$$

Chapter 13 One-Way ANOVA
What do we use ANOVA for?

- Calculates the difference among more than 2 population means
- Analysis Of Variance → analyze the difference among the group means, not their variances
- Within group variation: measures random variation
- Among-group variation: due to difference from group to group

Total Variation (SST) = Among-Group Variation (SSA) + Within Group Variation (SSW)

df = n -1 df = c – 1 df = n – c

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ANOVA Hypothesis for the means

Ho: $\mu_1 = \mu_2 = \mu_3 \dots \mu_c$ Ha: Not all μ_j are equal (where j = 1,2,3…c)

ANOVA F Test

- ANOVA is conducted to find the F_{STAT} for 3 or more groups of samples
- **<u>Assumptions:</u>** Randomness & independence, normality, and homogeneity of variance (equal variance)
- It is right skewed, and we conduct the upper tail test, reject H_0 if $F_{STAT} > F_\alpha$
- First, we look at Levene's Test, then we conduct ANOVA F-Test. Then, we analyze "Post-Hoc" to determine which μ_j is different
- **Ho:** $\sigma_1^2 = \sigma_2^2 = \sigma_3^2 \dots = \sigma_c^2$ (equal variance)
- **Ha:** At least one σ_j^2 is different (unequal variance)

ANOVA Summary table:

C = # of groups n = total sample size = c x observations in each group
For example: 4 groups with 8 values, n = 4 x 8 = 32

Sources	df	Sum of sq.	Mean of Sq. (var)	F STAT
Among Groups (n:df)	**c-1** = 4 – 1 = 3	SSA = 240	MSA = 240 / 3 = 80	F=MSA/MSW = 80 / 20 = 4
Within Groups (d:df)	**n-c** = 32 - 4 = 28	SSW = 560	MSW= 560/28 = 20	
Total	**n-c+ c-1** = **n-1** = 31	SST =SSA+SSW =800		

Relations in the table above

SSA + SSW = SST $MSA = SSA / (c – 1)$ $MSW = SSW / (n – c)$
$MST = SST / (n – 1)$ $F_{STAT} = MSA / MSW$

Levene's Test:

- To test the assumption of mean, Levene's test will be given
- The data should have equal variance in order to compute ANOVA F Test
- **Sig = p-value**, compare p-value with α, to make the decision

p-value ≥ α, do not reject H_0 **, they all have equal variance , ($F_{STAT} \approx 1$)**
p-value < α, reject H_0 **, not all the variances are equal, ($F_{STAT} > 1$)**

Example →

Independent Samples Test

		Levene's Test for Equality of Variances		t-test for Equality of Means						
									95% Confidence Interval of the Difference	
		F	Sig.	t	df	Sig. (2-tailed)	Mean Difference	Std. Error Difference	Lower	Upper
DVWORI	Equal variances assumed	1.493	.257	2.887	8	.020	2.0000	.6928	.4024	3.5976
	Equal variances not assumed			2.887	6.817	.024	2.0000	.6928	.3528	3.6472

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Using the calculator to do an ANOVA, given data →

A	B	C
2	5	5
4	6	8

Hypothesis:
Ho: $\mu_1 = \mu_2 = \mu_3$
Ha: Not all μ_j are equal (where j = 1,2,3)
!! (DO NOT WRITE IT AS $\mu 1 \neq \mu 2 \neq \mu 3 \ldots \mu c$)

Calculator Instructions:

1) Input all group/factor name numbers (repeat as needed) in a naming List 1, then input all data-values accordingly in List 2 (see the table on the right)
2) STAT →TEST→ ANOVA → How Many? 1 (always) → Factor A: List 1 (the naming list) → Dependent: List 2 (the observations data list) → EXE
3) The results such as F_{STAT} and p-value are computed.

List 1 – FactorA	List 2 – Dependent
1	2
1	4
2	5
2	6
3	5
3	8

Fining F Critical Value: 1). STAT → DIST → F → InvF; 2). Area: α
3). n:df = (c – 1) d:df = (n – c)

After conducting ANOVA F test:
Reject Ho → use POST-HOC to determine which μ_j is different
Do not reject → You're done!

Post-Hoc

- Known as the Tukey-Kramer Procedure for multiple comparison
- Post Hoc Data will be given, you just need to analyze it
- Helps determine which μ_j is different
- To analyze Post Hoc, compare the p-value with α

Example:
α = 0.05

Groups	Mean Difference	Sig.
A compared to B	13	0.029
A compared to C	5	0.318
B compared to C	1	0.118

Comparison –

H_0 μA = μB H_1 μA ≠ μB	p-value: 0.02 < α: 0.05 → reject H_0 , so μA ≠ μB
H_0 μA = μC H_1 μA ≠ μC	p-value: 0.03 < α: 0.05 → reject H_0 , so μA ≠ μC
H_0 μB = μC H_1 μB ≠ μC	p-value: 0.188 > α: 0.05 → do not reject H_0 , so μB = μC

Analysis – μA is the different since it doesn't equal to μB = μC
(*If another case results give you that μA < μB and μA > μC, then μC < μA < μB)

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Chapter 14 Chi-Square Tests (χ^2 , is pronounced as [kaɪ] square)
Chi-Square Test is for:

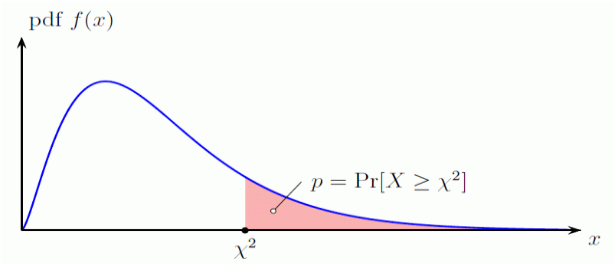
- Testing the difference between 2 or more population proportions
- Whenever we are given categorical responses, we use a *contingency table*

	Column Variable (Group)		
Row Variable	1	2	Totals
Items of interest	X_1	X_2	X
Items not of interest	$n_1 - X_1$	$n_2 - X_2$	n - X
Totals	n_1	n_2	n

- The example 2 x 2 contingency table can display numbers or percentage

Right skewed distribution with a right/upper tail test, **Reject H_0 if $\chi^2_{STAT} > \chi^2_a$**

The χ^2_{STAT} test statistic approximately follows a chi-square distribution with (r-1) x (c-1) degree of freedom for rows x columns contingency tables



Computing the Estimates Overall Proportion for 2 Groups:

$$\bar{P} = \frac{x1+x2}{n1+n2} = \frac{X}{n}$$

To test two-sample population proportion, we can use 2-P under Z Test for
H_0: $\pi_1 = \pi_2$, $\pi_1 \leq \pi_2$, $\pi_1 \geq \pi_2$ **H_1: $\pi_1 \neq \pi_2$, $\pi_1 > \pi_2$, $\pi_1 < \pi_2$**
And Chi-Square Test only for **H_0: $\pi_1 = \pi_2$** **H_1: $\pi_1 \neq \pi_2$**

Chi-Square (2 Groups) Test Calculator Instructions:

STAT - TEST – CHI - 2Way - Observed: Mat A - >MAT – Mat A –
DIM: set up your rows/columns for both Mat A and Mat B before anything
m = # of rows **n = # of columns**
- Mat A: only enter values here!! Enter values from the contingency table
- EXIT x2 - Go down to “Expected: Mat B”, press EXE
The calculator shows the x^2-value, p-value, and the degree of freedom
- Press Mat at the bottom right → Mat B → EXE
- Obtain new values (only f_e values)

Critical Value Calculator Instructions:

STAT - DIST → Chi → InvC – Area = α - df: obtain from test result - EXE (Since it is a right tail test, there is only 1 value and it is a positive number)

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Chi-Square Test for Difference Among 3 or More population Proportions:

Ho: $\pi_1 = \pi_2 = \pi_3 \ldots \pi_c$

Ha: Not all π_j are equal (where j = 1,2,3…c)

The contingency table is rows x columns

Calculator instruction for Chi-Square (more than 2) Test and for Critical Value are the same as above

Chi-Square Test of Independence

- To test the independence of two categorical variables
- Right skewed; right/upper tail test
- Almost the same as x^2 test in terms of decision rules, but the hypothesis and conclusion are different

  H_0: The two categorical variables are independent (there is no relationship between them)

  H_a: The two categorical variables are dependent (there is a relation between them)

Equation: $\chi^2_{\text{STAT}} = \sum_{all\ cells} \frac{(f_o - f_e)^2}{f_e}$

Expected Frequency:

f_e = (Row Total x Column Total) / n

Row total = sum of the frequencies in the row

Column total = sum of the frequencies in the column

Degrees of Freedom = (r – 1) (c – 1)

OR you can use the calculator instruction above to conduct the x^2 test and to find the χ^2 critical value

Condition: are all the expected frequency (f_e) ≥ 5 where f_e = (row total x column total)/(row total or total)

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