Sales and Service Performance in Bank Branches with Non-Volume Related Activities

Wade D. Cook and Moez Hababou

Schulich School of Business
York University, Toronto
Canada, M6J 1P3

August 24, 2000
Abstract

Studies of bank branch performance have, to date, concentrated on obtaining a single perspective of efficiency. As the financial services industry has intensified, banks have increasingly engaged in a proactive, differentiated and customer-based strategy in retail banking in which the sales component of the bank branch activity is emphasized. With the emerging sales culture within banks, as discussed earlier, there is a need to evaluate both sales and service performance. Cook, Hababou, and Tuenter (1999) have proposed a model to evaluate simultaneously the sales, service, and aggregate efficiencies of a bank branch. This model accounted for the fact that inputs, in particular resources, are often shared among these functions. In this paper, extending the DEA additive model, we derive the various optimal efficiency scores while taking into account non-volume related activities, that is those involving resources that can not be assigned to a specific input or output. Again, the proposed model recommends the optimal split of the shared resources that maximises the aggregate efficiency.

Key words: Data Envelopment Analysis, Sales, Service, Shared resources, Banking, Efficiency, Non-volume related activities.
1 Introduction

The study of performance within banks has been the focus of considerable research over the past two decades. One of the earliest studies, due to Sherman (1984) and directed specifically at the branch level, used data envelopment analysis (DEA) to derive a measure of efficiency for each branch relative to others in the comparison set. Developed by Charnes et al. (1978), DEA views branches in terms of a set of inputs and outputs. Numerous studies have followed Sherman’s, including Parkan (1987), Oral et al. (1991), Sherman and Ladino (1995), Berger et al (1991) and (1993), and others. More recently, Schaffnit et al (1997) completed an in depth study of operational performance within a major Canadian bank. Oral, Kettani, and Yolalan (1992) used DEA to measure the relative efficiency of a network of 44 commercial branches in Turkey. Barr and Siems (1994) used DEA to produce a bank failure prediction model called CAMEL. Other studies have focused on the more complex issue of measuring the allocative efficiency of banks by evaluating how strategy, process, and people are efficiently aligned (Frei et al 1997). Cook, Hababou, and Tuenter (1999) proposed a model to evaluate simultaneously service and sales efficiencies of bank branches. See Berger and Humphrey (1997) for a comprehensive review of past efficiency studies to financial institutions in various countries.

Such studies typically concentrate either on performance in the operational (work volume) sense, or else view bank branches as intermediaries, hence derive a profitability score for each unit. With the former, outputs are generally a set of service transactions such as numbers of deposits, account openings, and so on, while inputs are typically resources -- staff, operating expenses, etc. The latter, intermediary framework utilizes as outputs profitability variables, namely, returns on investments and loans, earned interest on customer accounts, and fees; inputs would generally be the same as in the operational efficiency model. Several variations on these two structures have been examined.

The financial services industry has been undergoing a major revolution in recent years due to deregulation, desintermediation, globalization, and technological progress. These forces have combined to cause an increased competition between various financial institutions and traditional boundaries between the businesses of each group of financial institutions to blur. Among the financial institutions whose businesses have evolved the most dramatically during the last decade, banks are certainly among the most visible ones. Banks face today an intense, global, dynamic and fast paced competition which is seriously leading them to rethink of the nature of their role in the society in light of the formidable regulatory and technological changes. These changes affect the way services are provided, the channels used to deliver those services as well as the very nature of financial services providers.

Banks are presently evolving from their traditional role as reactive monetary intermediaries and service providers toward a more general and proactive function as universal financial agents with a distinct sales culture. Banks have increasingly engaged in proactive, differentiated and customer-based strategy in retail banking in which the sales component of the bank branch activity is emphasized: "In the future, the banking branches that do succeed will likely function more as sales centers than service outlets".¹

A major challenge for banks is, thus, to create a customer-centric culture. Financial services have been historically very slow and resistant to changes mainly due to their heavy bureaucratic structures. This behavior was not a serious problem while customers had little financial initiative and the competitive landscape was relatively uncrowded. The situation today, however, is quite different as consumers are continuously being offered an array of new financial products from both traditional and non-traditional firms, which requiring banks to develop a new expertise in marketing, sales and customer services. Consumers have kept pace by demanding customized products and fast response to their financial needs. Deloitte and Touche (1995) revealed that branch staff lack sales and financial advisory skills. Beyond automation of basic transactions, financial institutions need to change themselves from transaction-oriented

organizations into sales and service organizations. It is, therefore, a must to analyze bank performance while incorporating both the sales and the service perspectives. To do this, it is more appropriate to study efficiency at the branch level. As Berger, Leusner and Mingo (1997) observed, analyses at the branch level can be more appropriate than analyses at the bank level because: (i) bank efficiency studies just reveal the average efficiency estimate for all branches but fail to uncover potential discrepancies and inequalities in efficiency among branches; (ii) branch analysis helps identifying potential targets for audits for the purpose of mapping out best practices; and (iii) managerially speaking, branch analysis can be more fruitful by recommending practical solutions such as consolidating branches in certain geographical region, replacing an inefficient local manager or investigating the possible efficiency effects of a potential merger.

Studies of bank branch performance have, to date, concentrated on obtaining a single perspective of efficiency. Performance measurement using tools such as Data Envelopment Analysis (DEA) as proposed by Charnes, Cooper and Rhodes (1978) has tended to concentrate on achieving a single measure for each member of a set of decision making units (DMUs). In most applications, a single measure of production or profit efficiency, provided by the DEA methodology has been an adequate and useful means of comparing units and identifying best performance. This has been particularly true in the case of banks, where the primary candidates for DMUs are branches, and in the traditional setting, product and prices have tended to be undifferentiated. With the emerging sales culture within banks, as discussed earlier, there is a need to evaluate both sales and service performance. This will permit management to identify those units who are on strategy, in the sense of promoting the sales mandate of the organization. As well, sales performance standards can be established that are sensitive to differences in environmental settings, for example, different demographic, regional, and economic situations. At the same time, a measure of branch efficiency relative to the more routine service or transaction function must, as well, be estimated. There is then a need to develop a methodology for deriving separate measures of branch performance for the sales and service functions.

A number of difficulties arise in developing such measures with the principal one being the issue of shared resources; resources such as staff and operating budgets are split between the sales and service functions of the branch. No mechanisms are currently in place, however, to track the percentage of usage of any given resource as it applies to a particular function. Some banks do attempt to estimate the range of the percentage share of a resource allocated to each of the sales and service components.

A second difficulty involves bounds on factor multipliers. With transaction or volume-related tasks, time study data provides for limits that can be translated into cone ratio bounds in the sense of Charnes et al (1990). Thus, for the service function, one can impose constraints that render the efficiency measurement exercise much more pragmatic than might be the case if no limits existed. For much of the sales related work within branches, however, there are no such data available.

The modeling of shared resources within the DEA structure has been examined previously. Beasley (1995) has developed a non-linear model for capturing teaching and research performance when inputs such as available budgets must be split between the sets of outputs defined by these two functions. Cook et al. (1998) have looked at a similar problem involving service transactions and (volume-related) sales transactions, where bounds on transaction times were available in both cases. They demonstrate that by permitting a different set of multipliers \( \mathbf{v} \) on that portion of the resources allocated to service activities than the multipliers \( \mathbf{v} \) on sales activities, a linear model results.

In this chapter, we view sales can as consisting of two components: (1) activities involving customer interaction (for example portfolio reviews) that may or may not result in a sale; and (2) the transaction work after a sale has been negotiated. The former is referred to as "non volume related," while the latter is "volume related" activity. We are thus interested in total sales activities, both volume related and non-volume related, not simply the transaction (for example, routine) portion of sales as examined in Cook, Hababou, and Tuenter (1999).

In this paper we develop a modification of the standard DEA structure which yields both a sales and a service measure. For convenience, we adopt the additive DEA model structure of Charnes et al (1985). In
Section 4.2 we demonstrate that this model can be viewed as a generalization of the standard radial models. Alternatively, the standard radial models are shown to be special cases of, or, at least, constrained versions of the general additive model. In Section 4.3 this general structure is used to formulate a dual-component measure to capture both sales and service functions in a bank branch setting. Section 4.4 uses a sample of branches from a Canadian bank to illustrate the model. Concluding remarks appear in Section 4.5.

2 The General Additive Model

In the next section we develop a dual-component DEA model for evaluating both sales and transaction functions within bank branches. For purposes of that development, the Pareto-Koopmans, or additive model, structure is exploited. While the additive model is seldom the structure of choice in most DEA analyses (one generally utilizes one of the radial models), it will be demonstrated that its structure is, in fact, a general framework containing the radial models as special cases. Specifically, any of the standard models are obtainable by way of constrained versions of the additive model. For development purposes herein, it is convenient to approach the standard models from this angle, rather than in the more conventional way. It is instructive to examine both dual and primal forms of the additive model, namely:

The Dual (D1)

\[
\text{Min } \sum_{i} \nu_{i} x_{io} - \sum_{r} \mu_{r} y_{ro} - \mu_{0} \quad (2.1a)
\]

Subject to:

\[
\sum_{i} \nu_{i} x_{ij} - \sum_{r} \mu_{r} y_{ij} - \mu_{0} \geq 0, \quad \forall j \quad (2.1b)
\]

\[
\mu_{j} \geq \frac{y_{jo}}{y_{ro}}, \quad \forall r \quad (2.1c)
\]

\[
\nu_{i} \geq \frac{y_{io}}{y_{ro}} \quad \forall i \quad (2.1d)
\]

It is noted that we have chosen lower bounds on the multipliers (2.1c) and (2.1d) that are DMU-specific. This is usually referred to as the units invariant form of the model. The "dual" of (D1) is the model (P1):

The Primal (P1)

\[
\text{Min } \sum_{i} (s^{1}_{i} / x_{io}) + \sum_{r} (s^{2}_{r} / y_{ro}) \quad (2.2a)
\]

Subject to:

\[
\sum_{j} \lambda_{j} x_{ij} + s^{1}_{i} = x_{io}, \quad \forall j \quad (2.2b)
\]

\[
\sum_{j} \lambda_{j} y_{ij} - s^{2}_{r} = y_{ro} \quad \forall r \quad (2.2c)
\]

\[
\sum_{j} \lambda_{j} = 1 \quad (2.2d)
\]

\[
s^{1}_{i}, s^{2}_{r}, \lambda_{j} \geq 0 \quad \forall i, r, j \quad (2.2e)
\]

If we adopt the notation

\[
\Theta_{i} = 1 - s^{1}_{i} / x_{io}, \Phi_{r} = 1 + s^{2}_{r} / y_{ro}
\]

and let \(\Theta_{i} = 1 - \Theta_{i}\) and \(\Phi_{r} = 1 - \Phi_{r}\), model (P1) becomes
The Primal (P2)

\[
\begin{align*}
\text{Min} \quad & \sum_i \bar{\Theta}_i + \sum_r \bar{\Phi}_r \\
\text{Subject to :} \quad & \\
\sum_i \lambda_j x_{ij} + \bar{\Theta}_r x_{io} = x_{io}, \quad & \forall j \\
\sum_r \lambda_j y_{rj} - \bar{\Phi}_r y_{ro} = y_{ro}, \quad & \forall r \\
\sum_j \lambda_j = 1 \\
\bar{\Theta}_i, \bar{\Phi}_r, \lambda_j & \geq 0 \quad \forall i, r, j
\end{align*}
\] (2.4a, 2.4b, 2.4c, 2.4d, 2.4e)

This format is a particularly convenient way to view the additive model, as it exhibits an immediate connection to other models. This form is related to the "Russell Measure" as discussed in Färe and Lovell (1978). There, the objective function takes the form

\[
\begin{align*}
\text{Min} \quad & \left[ \sum_i \Theta_i + \sum_r (1/\Phi_r) \right] / (I + R)
\end{align*}
\]

where I, R are the numbers of inputs and outputs respectively. Thrall (1996) and Park, Cooper, and Pastor (1999) discuss several variations on the additive model. It is immediately clear that one can adopt a purely input oriented variation on the additive model concept, by setting \( \Phi_r = 0 \) for all \( r \), and replacing constraints (2.4b) and (2.4c) by

\[
\begin{align*}
\sum_i \lambda_j x_{ij} + \bar{\Theta}_r x_{io} & \leq x_{io}, \\
\sum_r \lambda_j y_{rj} & \geq y_{ro}
\end{align*}
\] (2.5a, 2.5c)

This type of structure is discussed in Zieschang (1984). Furthermore, if we restrict the \( \bar{\Theta}_j \) further by requiring that they all be equal, then we have a structure equivalent to the standard input oriented radial model of Charnes, Cooper and Rhodes (1978).

In the case that the input oriented approach is to be taken, in which case (2.5a) and (2.5b) replace (2.4b) and (2.4c) in the primal problem (P2), the equivalent modification to the dual problem (D1) is to replace the lower bound on \( \mu_r \) (constraint (2.1c)) by

\( \mu_r > 0 \).

As with the Russell measure, an appropriate measure of performance in the input oriented additive model is

\[
R_i = \sum_{j=1}^{J} (1 - \bar{\Theta}_j) / I = \sum_{j=1}^{J} \Theta_j / I
\] (2.6)

It is noted that in the restricted case where \( \Theta_j = \Theta \) for all \( i \) (the BCC radial model), \( R_i = \Theta \). In any event, it will be the case that \( 0 \leq R_i \leq I \) with \( R_i = 1 \) if all \( \bar{\Theta}_j = 0 \); for example, in this case the pair \((Y^0, X^0)\) is on the frontier or an extension.
Stated formally then, the pure input version of (P2) is (P2'):

\[
\begin{align*}
\text{Min} & \quad \sum_i \bar{\theta}_j / I \\
\text{Subject to:} & \\
\sum_j \lambda_j x_{ij} + \bar{\theta}_j x_{io} &= x_{io}, & \forall i \\
\sum_j \lambda_j y_{ij} - \bar{\phi}_j y_{io} &= y_{io}, & \forall r \\
\sum_j \lambda_j &= 1 \\
\bar{\theta}_j, \lambda_j &\geq 0 & \forall i, j
\end{align*}
\]

Thus, the additive model can be viewed as a flexible mechanism for capturing different aspects of efficiency. Admittedly, restricted versions of the model can fail to be "comprehensive" in the sense discussed by Park et al (1999). Obviously, it will be true that restricting attention to the input side of the problem, for example, can mean that improper envelopment can occur, as is well known in the radial models.

3 An Additive Model for Sales and Service Measures

In this paper, product volumes are used as the tangible outputs or outcomes resulting from sales efforts within branches. For the sales components of branch activities, these products would include account openings for RRSPs and mutual funds, letters of credit, collateral and non collateral loans, mortgages, and so on. While one may view branch performance from the perspective of the profitability from these products, we choose herein to concentrate on operational efficiency. Hence, it is the number or volume of transacted sales (for example, number account openings) and not their monetary values which is of interest.

With service activities (for example, deposits, withdrawals, etc.), resource usage can generally be linked directly to the specific volume or product involved. For example, a teller's time is spent servicing customer transactions, wherein those customers have requested services pertaining to given products. The specifications for those products and the related service requirements are generally well defined. Moreover, reasonably accurate estimates of service times per unit of output of a product are obtainable via time studies.

However, as indicated earlier, in the case of the sales component, one must view activities as consisting of two parts. First, there are non-volume-related activities of sales staff, for example, those that are not linked to any specific product. Such activities would include responding to customer queries, routine tasks such as reproduction of forms, reviewing customer portfolios, carrying out computer searches, and so on. Support costs for print materials, computer expenses, etc. would, as well, fall into this category. The second set of activities that make up the unit time involvement surrounding the sale of a product consists of those tasks linked directly to that product, after the decision to purchase is made. These activities would include preparation tasks for the product at hand (the filing of documents, preparation of certificates, etc.). Unlike the non-volume related activities, these latter tasks are well understood and are characterized by known time estimates arrived at in the same manner as is the case for service activities. We define the following notation for the dual component model:

**Notation**

**Indices**

\((i,r,j)\)- index on (inputs, outputs, DMU)
Inputs

- \( x^1_j \) - vector of dedicated service inputs for DMU \( j \)
- \( x^2_j \) - vector of dedicated sales inputs for DMU \( j \)
- \( x^s_j \) - vector of shared inputs for DMU \( j \)

Outputs

- \( y^1_j \) - vector of service outputs for DMU \( j \)
- \( y^2_j \) - vector of sales outputs for DMU \( j \)

Input Multipliers

- \( \nu^1 \) - multipliers of \( X^1_j \)
- \( \nu^2 \) - multipliers of \( X^2_j \)
- \( (\nu^1, \nu^2) \) _ multipliers on the (service, sales) portion of \( x^s_j \)

Output Multipliers

- \( \mu^1 \) - multipliers measuring the per unit processing time for service outputs
- \( (\mu^2, \mu^{22}) \) - multipliers measuring the per unit processing times for the (volume, non-volume) related portions of sales outputs \( y^2_j \).

To model the dual component performance we assume that inputs fall into three groups: (i) inputs \( x^1_j \) dedicated to the service function and which aid in generating service transactions; (ii) inputs \( x^2_j \) dedicated to the generation of sales; and (iii) \( x^s_j \), those inputs that are shared between the two functions. The first group might, for example, include counter staff, the second sales staff, and the third support staff of various types. In some settings it might be argued that all staff share the service and sales roles, meaning that \( X^1 \) and \( X^2 \) are non existent. Let \( I^1, I^2, I^s \) denote the dimensions of the three input vectors.

Let \( \alpha \) (0 ≤ \( \alpha \) ≤ 1) denote the portion of shared resource \( x^s_{ij} \) allocated to the service function of DMU \( j \) (the remainder (1-\( \alpha \)) of \( x^s_{ij} \) allocated to sales). Here, \( \alpha \) is a decision variable at the disposal of each DMU. We assume that different multipliers \( \nu^1 \) may be chosen for those shared resources allocated to the service function than those \( (\nu^2) \) allocated to sales. This is an important feature in our model, and differs from the model for shared resources proposed by Beasley (1995) in evaluating teaching and research functions in universities. This recognizes that if support staff and technology, for examples, are shared resources, then their relative importance in terms of service transactions may be different than that for sales. Let us represent the aggregate virtual input for DMU \( j \) (weighted input) by

\[
\nu^1 X^1_j + \nu^1 (\alpha X^1_j) + \nu^2 ((1-\alpha) X^2_j) + \nu^2 X^2_j
\]

Here, \( \alpha X^1_j \) denotes the vector \((\alpha x^1_{ij}, \alpha x^1_{ij}, \ldots, \alpha x^1_{ij})\). The term (1-\( \alpha \))\( X^2_j \) is defined accordingly. The aggregate output for the DMU is given by \( \mu^1 Y^1_j + \mu^{22} Y^2_j \).

The component inputs and outputs are given by

<table>
<thead>
<tr>
<th>Service ⇒</th>
<th>Input ⇒</th>
<th>Output ⇒</th>
</tr>
</thead>
<tbody>
<tr>
<td>Service ⇒</td>
<td>( \nu^1 X^1_j + \nu^1 (\alpha X^1_j) )</td>
<td>( \mu^1 Y^1_j )</td>
</tr>
</tbody>
</table>
Utilizing the usual terminology of the additive model, the expression for the aggregate objective function for DMU "o" in the dual problem would be:

\[ e_{ao} = (S_e I_o + S_a I_o) - (S_e O_o + S_a O_o) - \mu_o \]  

(3.1)

In deriving appropriate measures of both service and sales performance for a given branch “o” the principal issue to be resolved is that of what objective function to choose. Arguably, one could minimize \( e_{ao} \), hence deriving multipliers \((\mu^e, \nu^e)\) for which the overall performance of the branch is rated as high as possible. Here, this means \( e_{ao} \) being as close to zero as possible. A possible danger in this approach is that a minimal \( e_{ao} \) may be achieved by reducing one of the component measures as much as possible without regard for the size which the other, complementary, measure might take. Thus, the service measure could be driven down close to 0 (\( \theta = 100\% \)), while the sales measure could end up being inordinately large. An alternative to optimizing the aggregate measure \( e_{ao} \) (expression (3.1)), is to attempt to optimize, in some manner, both the service measure

\[ e_s^1 = v^1 X_s^1 + v^1 (\alpha X_s^1) - \mu^1 Y_s^1 - \mu^1_o \]  

(3.2)

and sales measure\(^2\)

\[ e_o^2 = v^2 ((1-\alpha) X_o^2) + v^2 X_o^2 - \mu^2 Y_o^2 - \mu^2_o \]  

(3.3)

One approach is to minimize the maximum inefficiency. For example, we solve the goal programming problem.

\[ Min \ d \]

subject to

\[ e_s^1 \leq d, \ e_o^2 \leq d \]

\[ S_e I_j - S_a O_j \geq 0, \ S_a I_j - S_e O_j \geq 0 \quad \forall j \]

In attempting to reduce the maximum inefficiency, the model has the tendency to equalize the sales and service performance measures \( d \) if feasibility permits. In some respects this could be justified insofar as one can argue that a branch will or should give equal importance to all components of its business. Formally, the dual form of the proposed model is given by

\(^2\) In the context of the VRS structure, we let \( \mu^1, \mu^2 \) denote service and sales variables, respectively.
Min $d$

Subject to

$-v^1 X^1_o - v^1 (\alpha X^1_i) + \mu^1 Y^d + \mu^1_o + d \geq 0$

$v^1 X^1_j + v^1 (\alpha X^1_j) - \mu^1 Y^d - \mu^1_j \geq 0 \quad \forall j$

$-v^{s2} ((1-\alpha) X^2_o) - v^2 X^2_o + \mu^1 Y^d + \mu^1_o + d \geq 0$

$+v^{s2} ((1-\alpha) X^2_j) + v^2 X^2_j - \mu^2 Y^2 \geq 0 \quad \forall j$

$v^1_i \geq \frac{1}{x^d_i} \quad \forall i \in I_i$

$v^2_i \geq \frac{1}{x^d_i} \quad \forall i \in I_2$

$v^{s1}_i \geq \frac{1}{x^d_i} \quad \forall i \in I_i$

$v^{s2}_i \geq \frac{1}{x^d_i} \quad \forall i \in I_i$

$\mu^1_i \geq 0$

$\mu^2_i \geq 0$

Note that we have introduced the lower bounds $1/x^d_i |I_i|$, etc. to force $0 \leq d \leq 1$. Here, $|I_i|$ denotes the cardinality of the input set $I_i$. To solve for the non linearity created by the products $\alpha v^{s1}$ and $(1-\alpha)v^{s2}$, we introduce the change of variables

$\alpha v^{s1} = v^{s1} \quad \text{and} \quad (1-\alpha)v^{s2} = v^{s2}$

Then, replace the two constraints $v^{s1} \geq 1/x^d_o$ and $v^{s2} \geq 1/x^d_o$ by $\alpha v^{s1} \geq \alpha / x^d_o$ and $(1-\alpha)v^{s2} \geq (1-\alpha) / x^d_o$. It is generally the case that constraints will be imposed on the $\sim \alpha$; specifically, the percent of any resource that can be allocated to the service component will be required to be within some interval, namely

$L^1_i \leq \alpha_i \leq L^2_i$

Model (D2) can now be rewritten in the form of (D3):
**Dual (D3)**

**Min** \( d \)

Subject to

\[
\begin{align*}
&-v_1^1 x_o^1 - \nabla v^1 x_o^1 + \mu^1_y^d + \mu^1_y^j + d \geq 0 \\
&v_1^j x_j^1 + \nabla v^1 x_j^1 - \mu^1_y^d - \mu^1_y^j \geq 0 & \forall j \\
&-v_2^1 x_o^2 - \nabla v^2 x_o^2 + \mu^1_y^d + \mu^1_y^j + d \geq 0 \\
&+v_2^j x_j^2 + \nabla v^2 x_j^2 - \mu^2_y^d - \mu^2_y^j \geq 0 & \forall j \\
&v_i^1 \geq \frac{1}{n} \| I_i \|, & \forall i \in I_1 \\
&v_i^2 \geq \frac{1}{n} \| I_i \|, & \forall i \in I_2 \\
&v_i^{s1} - \alpha v_i^{s2} \geq 0, & \forall i \in I_1 \\
&v_i^{s2} - \alpha v_i^{s1} \geq 0, & \forall i \in I_1 \\
&\mu_i^1 \geq 0 \\
&\mu_i^2 \geq 0 \\
&L_i \leq \alpha \leq L_i
\end{align*}
\]

The dual of this problem is given by (P3):

**Primal (P3)**

\[
\begin{align*}
&\text{Max} \ p = \left\{ \sum_{i \in I_1} (s_i^1 / x_{io}^1 I_i^1) + \sum_{i \in I_1} (s_i^2 / x_{io}^2 I_i^2) \right\} + \sum_{i \in I_1} (s_i^{s1} / x_{io}^1 I_i^1) \\
&+ \sum_{i \in I_1} (s_i^{s2} / x_{io}^2 I_i^2) + L_i^1 \gamma_i^1 - L_i^2 \gamma_i^2 \}
\end{align*}
\]

Subject to:

\[
\begin{align*}
&\sum_j \lambda_{ij} x_{ij}^1 - \lambda_{n+i} x_{io}^1 + s_i^1 \leq 0, & \forall i \in I_1 \\
&\sum_j \lambda_{ij} x_{ij}^1 - \lambda_{n+i} x_{io}^1 + s_i^{s1} \leq 0, & \forall i \in I_1 \\
&\sum_j \lambda_{ij} x_{ij}^2 - \lambda_{n+i} x_{io}^2 + s_i^2 \leq 0, & \forall i \in I_2 \\
&\sum_j \lambda_{ij} x_{ij}^2 - \lambda_{n+i} x_{io}^2 + s_i^{s2} \leq 0, & \forall i \in I_1 \\
&\sum_j \lambda_{ij} y_{ij}^1 + \lambda_{n+i} y_{io} \leq 0, & \forall r \in R_1 \\
&\sum_j \lambda_{ij} y_{ij}^2 + \lambda_{n+i} y_{io} \leq 0, & \forall r \in R_2 \\
&\lambda_{n+i} + \lambda_{n+i+1} = 1 \\
&s_i^{s1} / (x_{io}^1 I_i^1) + s_i^{s2} / (x_{io}^2 I_i^2) + \gamma_i^1 - \gamma_i^2 \leq 0, \\
&\gamma_i^1, \gamma_i^2, \lambda_{ij}^1, \lambda_{ij}^2, s_i^1, s_i^2, s_i^{s1}, s_i^{s2} \geq 0 & \forall i, j
\end{align*}
\]
Letting $\theta^i = \frac{y}{x_i}$, $\theta^{2i} = \frac{y}{x_i}$, $\theta^s = \frac{y}{x_s}$, $\theta^{2s} = \frac{y}{x_s}$, problem (P3) can be written as (P4)

**Primal (P4)**

$$Max \; e_p = \left\{ \sum_{i \in I_1} (\theta^i | \mu|) + \sum_{i \in I_2} (\theta^{2i} | \mu|) + \sum_{i \in I_s} (\theta^s | \mu|) \right\}$$

Subject to:

$$\sum_{i \in I_1} \lambda^i x^i - \lambda^i x^i + \bar{\theta}^i x^i \leq 0, \; \forall i \in I_1$$

$$\sum_{i \in I_2} \lambda^i x^i - \lambda^i x^s + \bar{\theta}^i x^s \leq 0, \; \forall i \in I_2$$

$$\sum_{i \in I_s} \lambda^i y^i + \lambda^i y^i \leq 0, \; \forall i \in I_s$$

$$\lambda^1 + \lambda^2 = 1$$

$$\bar{\theta}^i \; |\mu| + \bar{\theta}^{2i} \; |\mu| + \gamma^i - \gamma^{2i} \leq 0$$

$$\gamma^i, \gamma^{2i}, \lambda^i, \lambda^{2i}, \bar{\theta}^i, \bar{\theta}^{2i}, \bar{\theta}^s \geq 0 \; \forall i, j$$

It can be seen that (P4) is a direct generalization of (P2'). The equivalent of the $R_i$ measure given in (2.6) is

$$\bar{e}_p = 1 - e_p$$

It must be noted, of course, that $e_p = e_d$ (the objective function value of (D3)) is the maximum of the two components $e^i_p, e^{2i}_p$, as per (3.2) and (3.3). The separate sales and service measures would fall out as part of the analysis.

### 3.1 Application to Bank Branches

The model presented herein was applied to evaluating sales and service efficiency within a major Canadian Bank. As with most financial institutions the traditional role of the Bank as service provider has increasingly comprises a sales component. Data on twenty sample branches was selected and is displayed in Table 1. In this particular case a representative selection of outputs pertaining to service transactions and a set corresponding to sales products was chosen for demonstration purposes. Specifically, the outputs are:

**Service:**
- TOTMENU-total number of menu account transactions
- VISA-number of Visa, cash advances
- CAD-number of commercial deposit transactions

**Sales:**
- RSP-number of RSP account openings
- MORT-number of mortgages transacted
- BPL-number of variable rate consumer loans transacted

In the current example, inputs were restricted to personnel only. We have not included other operating expenses such as computers, rent, etc. Specifically, the inputs are:
Service: FSE-total number of full time equivalent service staff  
Sales: FSA-total number of full time equivalent sales staff  
Shared: FSU-total number of full time equivalent support staff  
FST-total number of full time equivalent "other" staff

Table 1 displays the data on all inputs and outputs for a sample of 20 branches of the bank. The results from applying model (D3) are shown in Table 2. Recall that \( d \) represents the maximum inefficiency associated with the two components (sales and service). The corresponding efficiency measures \( e_S \) (sales) and \( e_T \) (service) are displayed. It is noted that \( d = 1 - \min \{ e_S, e_T \} \). As noted earlier, this model tends to force \( e_T \) and \( e_S \) together and in a large percentage of the cases, the two measures are equal. We have not directly addressed the issue of an "aggregate" measure of efficiency which should be some combination of the two separate measures. Arguably, this aggregate measure \( e_A \) should be some average of the component scores. A reasonable candidate for this might be of the form

\[
e_A = \beta e_T + (1-\beta)e_S
\]

where \( \beta \) is the proportion of total resources consumed by the service component (dedicated service inputs together with shared inputs).

In this application of model (D3), the splitting variables \( \alpha_1 \) and \( \alpha_2 \) have each been restricted to the range \( .25 \leq \alpha \leq .75 \). This range would need to be established by branch consultants in much the same manner that ranges on output multiplies might be set by way of time study estimates.

### Table 1: Sales and Service Outputs and Inputs

<table>
<thead>
<tr>
<th>Transit #</th>
<th>TOTM</th>
<th>VISA</th>
<th>CAD</th>
<th>RSP</th>
<th>MORT</th>
<th>BPL</th>
<th>FSE serve</th>
<th>FSA sales</th>
<th>FOT</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>51803</td>
<td>2973</td>
<td>190522</td>
<td>421</td>
<td>567</td>
<td>101</td>
<td>55.25</td>
<td>48.07</td>
<td>59.55</td>
</tr>
<tr>
<td>2</td>
<td>10477</td>
<td>710</td>
<td>49898</td>
<td>75</td>
<td>172</td>
<td>13</td>
<td>14.73</td>
<td>32.39</td>
<td>17.15</td>
</tr>
<tr>
<td>3</td>
<td>11195</td>
<td>431</td>
<td>39523</td>
<td>68</td>
<td>73</td>
<td>19</td>
<td>9.6</td>
<td>14.22</td>
<td>8.55</td>
</tr>
<tr>
<td>4</td>
<td>6480</td>
<td>422</td>
<td>30713</td>
<td>49</td>
<td>48</td>
<td>15</td>
<td>10.48</td>
<td>9.05</td>
<td>5.15</td>
</tr>
<tr>
<td>5</td>
<td>37695</td>
<td>9211</td>
<td>26922</td>
<td>210</td>
<td>128</td>
<td>144</td>
<td>27.9</td>
<td>17.22</td>
<td>5.02</td>
</tr>
<tr>
<td>6</td>
<td>9211</td>
<td>362</td>
<td>43056</td>
<td>120</td>
<td>127</td>
<td>57</td>
<td>9.17</td>
<td>15.79</td>
<td>2.44</td>
</tr>
<tr>
<td>7</td>
<td>16483</td>
<td>529</td>
<td>13123</td>
<td>74</td>
<td>150</td>
<td>9</td>
<td>10.53</td>
<td>12.49</td>
<td>2.19</td>
</tr>
<tr>
<td>8</td>
<td>456</td>
<td>20</td>
<td>10127</td>
<td>6</td>
<td>29</td>
<td>15</td>
<td>10.6</td>
<td>10</td>
<td>5.45</td>
</tr>
<tr>
<td>9</td>
<td>5985</td>
<td>382</td>
<td>21945</td>
<td>32</td>
<td>28</td>
<td>19</td>
<td>8.71</td>
<td>8.02</td>
<td>3.94</td>
</tr>
<tr>
<td>10</td>
<td>8682</td>
<td>351</td>
<td>11010</td>
<td>84</td>
<td>78</td>
<td>52</td>
<td>7.05</td>
<td>11.25</td>
<td>2.36</td>
</tr>
<tr>
<td>11</td>
<td>5287</td>
<td>182</td>
<td>16474</td>
<td>59</td>
<td>97</td>
<td>15</td>
<td>6.61</td>
<td>9.42</td>
<td>1.88</td>
</tr>
<tr>
<td>12</td>
<td>18292</td>
<td>171</td>
<td>18014</td>
<td>104</td>
<td>84</td>
<td>443</td>
<td>7.3</td>
<td>9.33</td>
<td>1.79</td>
</tr>
<tr>
<td>13</td>
<td>5669</td>
<td>264</td>
<td>11303</td>
<td>36</td>
<td>62</td>
<td>144</td>
<td>3.96</td>
<td>3.92</td>
<td>1</td>
</tr>
<tr>
<td>14</td>
<td>9656</td>
<td>332</td>
<td>6745</td>
<td>65</td>
<td>63</td>
<td>14</td>
<td>6.7</td>
<td>8.62</td>
<td>0.92</td>
</tr>
<tr>
<td>15</td>
<td>18566</td>
<td>308</td>
<td>76174</td>
<td>134</td>
<td>80</td>
<td>14</td>
<td>10.29</td>
<td>10.94</td>
<td>4.87</td>
</tr>
<tr>
<td>16</td>
<td>39430</td>
<td>500</td>
<td>60832</td>
<td>199</td>
<td>199</td>
<td>200</td>
<td>25.07</td>
<td>20.55</td>
<td>4.69</td>
</tr>
<tr>
<td>17</td>
<td>11601</td>
<td>423</td>
<td>36692</td>
<td>73</td>
<td>137</td>
<td>107</td>
<td>12.25</td>
<td>10.91</td>
<td>3.88</td>
</tr>
<tr>
<td>18</td>
<td>8030</td>
<td>406</td>
<td>19598</td>
<td>62</td>
<td>86</td>
<td>50</td>
<td>9</td>
<td>13.35</td>
<td>3.16</td>
</tr>
<tr>
<td>19</td>
<td>16991</td>
<td>658</td>
<td>21334</td>
<td>91</td>
<td>111</td>
<td>78</td>
<td>12.7</td>
<td>18.02</td>
<td>2.11</td>
</tr>
<tr>
<td>20</td>
<td>10473</td>
<td>463</td>
<td>51225</td>
<td>132</td>
<td>71</td>
<td>39</td>
<td>18.15</td>
<td>18.65</td>
<td>6.92</td>
</tr>
</tbody>
</table>

### Table 2: Results from the model

<table>
<thead>
<tr>
<th>DMU</th>
<th>( \alpha_1 )</th>
<th>( \alpha_2 )</th>
<th>( d )</th>
<th>( e_S )</th>
<th>( e_T )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.742798</td>
<td>0.75</td>
<td>0.25</td>
<td>0.25</td>
<td>0.25</td>
</tr>
<tr>
<td>2</td>
<td>0.5</td>
<td>0.5</td>
<td>0.278917</td>
<td>0.5</td>
<td>0.5</td>
</tr>
<tr>
<td>3</td>
<td>70.14%</td>
<td>85.13%</td>
<td>80.24%</td>
<td>0.00%</td>
<td>23.59%</td>
</tr>
<tr>
<td>4</td>
<td>29.86%</td>
<td>14.87%</td>
<td>60.00%</td>
<td>19.76%</td>
<td>100.00%</td>
</tr>
<tr>
<td>5</td>
<td>29.86%</td>
<td>14.87%</td>
<td>60.00%</td>
<td>19.76%</td>
<td>100.00%</td>
</tr>
<tr>
<td>6</td>
<td>0.371086</td>
<td>0.746744</td>
<td>0.25</td>
<td>0.5</td>
<td>0.5</td>
</tr>
<tr>
<td>7</td>
<td>0.5</td>
<td>0.5</td>
<td>0.25</td>
<td>0.5</td>
<td>0.5</td>
</tr>
<tr>
<td>8</td>
<td>94.27%</td>
<td>34.77%</td>
<td>94.27%</td>
<td>34.77%</td>
<td>94.27%</td>
</tr>
</tbody>
</table>
4 Comments

This chapter has looked at modeling the efficiency of different components of bank branch operations; specifically sales and service. The technical difficulty encountered here is that some resources are shared between the sales and service functions. This has necessitated the introduction of variables that split shared resources between the components. A goal programming version of the additive DEA model is developed. By its very nature, this goal programming model has the tendency to force the component measures toward each other, thus assigning a branch equal measures of efficiency on sales and service in a number of cases. There is some argument that this property is quite realistic if one assumes that management would not be prone to emphasize sales at the expense of service and vice versa. Thus, the goal achievement idea appears to make practical sense. The conventional DEA approach has tended to concentrate on a single measure of performance for the DMU. Very often, however, there are multiple components or sub units within the DMU whose individual performance is required. The model provided herein provides a mechanism for developing multicomponent measures.

5 References


